Definite Integral

Let $f(x)$ be a continuous function over the interval $[a, b]$. Partition $[a, b]$ into n subintervals of width $\Delta x = \frac{b-a}{n}$ and endpoints $x_0 = a, x_1, ..., x_n = b$. Let $x_i^* \in [x_{i-1}, x_i]$. Then

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$$

Indefinite Integral

$\int f(x)dx = F(x) + C$ where $F(x)$ is an antiderivative of $f(x)$.

Fundamental Theorem of Calculus Pt 1

If f is continuous on $[a, b]$ then the function g such that

$$g(x) = \int_a^x f(t)dt \quad a \leq x \leq b$$

is continuous on $[a, b]$, differentiable on (a, b), and $g'(x) = f(x)$.

Fundamental Theorem of Calculus Pt 2

If f is continuous on $[a, b]$ then

$$\int_a^b f(x)dx = F(b) - F(a)$$

where $F(x)$ is an antiderivative of $f(x)$.

Properties

$$\int_a^b f(x)dx = -\int_b^a f(x)dx$$

$$\int_a^a f(x)dx = 0$$

$$\int_a^b cf(x)dx = c\int_a^b f(x)dx$$

$$\int_a^b (f(x) \pm g(x))dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$$

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

Common Integrals

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \text{ where } n \neq -1$$

$$\int \frac{1}{x} dx = \ln|x| + C$$

$$\int e^x dx = e^x + C$$

$$\int \cos(x) dx = \sin(x) + C$$

$$\int \sin(x) dx = -\cos(x) + C$$

$$\int \sec^2(x) dx = \tan(x) + C$$

$$\int \csc^2(x) dx = -\cot(x) + C$$

$$\int \sec(x)\tan(x) dx = \sec(x) + C$$

$$\int \csc(x)\cot(x) dx = -\csc(x) + C$$

$$\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin(x) + C$$

$$\int \frac{1}{1+x^2} dx = \arctan(x) + C$$

$$\int \tan(x) dx = \ln|\sec(x)| + C$$

$$\int \sec(x) dx = \ln|\sec(x) + \tan(x)| + C$$

Improper Integrals

Infinite Interval

$$\int_a^\infty f(x)dx = \lim_{t\to\infty} \int_a^t f(x)dx \qquad \int_{-\infty}^b f(x)dx = \lim_{t\to-\infty} \int_t^b f(x)dx$$

$$\int_{-\infty}^\infty f(x)dx = \lim_{s\to-\infty} \int_s^b f(x)dx + \lim_{t\to\infty} \int_a^t f(x)dx$$

*assuming BOTH limits on the right converge

Discontinuous Integrand

If f is continuous on $[a, b)$ and discontinuous at b then $\displaystyle\int_a^b f(x)dx = \lim_{t\to b^-} \int_a^t f(x)dx$.

If f is continuous on $(a, b]$ and discontinuous at a then $\displaystyle\int_a^b f(x)dx = \lim_{t\to a^+} \int_t^b f(x)dx$.

If f has a discontinuity at c where $a < c < b$ then

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

given that both of the integrals on the right converge.