A Hybrid Algorithm Solution for GPS Antenna Array

Cynthia Junqueira[1,2] , Moisés V. Ribeiro[1], João Marcos T. Romano[1],
Clodoaldo Lima[1], João Batista Destro-Filho[1]

State University of Campinas (UNICAMP)[1] - Campinas – SP- Brazil
Aerospace Technical Center – Aeronautical and Space Institute[2] (CTA/IAE) – São José dos Campos -Brazil
{Cynthia, mribeiro, romano, destro}@decom.fee.unicamp.br, moraes@dca.fee.unicamp.br

BIOGRAPHY

Cynthia Junqueira received the B.S. degree in Electrical Engineering from Faculty of Technology and Sciences and the M.Sc. degree from Aeronautic Technological Institute (ITA), Brazil, 1996. She is a researcher in Aerospace Technical Center (CTA/IAE), Brazil and since 1999, a Ph.D. student at UNICAMP. Her research interests include adaptive signal processing and antennas.

Moisés Vidal Ribeiro received the B.S. degree in Electrical Engineering from the Federal University of Juiz de Fora (UFJF), 1999. Since August 1999 Moisés is a M.Sc. student at UNICAMP. His interests are Linear and non-linear adaptive signal processing, Sub-band Signal Processing, neural networks, multimedia signal processing and DSP implementations.

João Marcos T. Romano received the B.S. and M.Sc. degree from UNICAMP and Ph.D. degree from University of Paris-XI in 1987. Since 1988, Dr. Romano joined the Communications Department of the Faculty of Electrical and Computer Engineering - UNICAMP, where he is now a full professor. He is the president of the Brazilian Communications Society for the 2000-2002 period.

Clodoaldo Lima received the M.Sc. degree in Electrical Engineering from UNICAMP in 2000. Now he is a Ph.D. student at UNICAMP. His research interests concern neural network and genetic algorithms.

J.B. Destro-Filho received the B. Eng. degree in 1991 and the M.Sc. degree in 1994 from the School of Electrical and Computer Engineering (FEEC), UNICAMP, Campinas, Brazil. He received the Ph.D. degree from the I3S Laboratory-CNRS, University of Nice-Sophia Antipolis, France, in 1998. Since 1999 he has been working as a postdoctoral researcher at the Communications Department, FEEC, UNICAMP, teaching undergraduate and postgraduate courses. His research interests include adaptive signal processing, high-order statistics, education and neuroscience.

ABSTRACT

This work addresses the applications of adaptive antenna arrays for GPS, in order to achieve a more accurate estimation of user position, as well as signal enhancement and interference cancellation.

A hybrid algorithm solution for adaptive antenna array is proposed, which is based on a constrained adaptive algorithm, associated with a multilayer perceptron neural network.

Two hybrid algorithms, using two kinds of constrained algorithms, are compared to each other and to the classical solutions. Simulations consider different realistic GPS situations, pointing out the effectiveness of the hybrid approach, in terms of lower computational burden, lower stead- state error and better radiation patterns with respect to the classical solutions.

INTRODUCTION

The Global Positioning System is important for a great variety of applications, including civil or military users. It enables real-time position, time and velocity accurate estimation, and possibility to use on a variety of platforms, 24 hours a day [1].

GPS signals are subject to several impairments, such as multipath, fading, tropospheric and ionospheric delays, power flutuactions due to scintillation, doppler effects, clock and receiver errors.

On the other hand, adaptive antennas may be considered as emerging techniques, which play an important role for mobile communications, for example. In fact, these devices are based on signal processing algorithms.

The application of adaptive antenna arrays for GPS is one of the ways in order to achieve a more accurate estimation of user position, as well as signal enhancement and interference cancellation. In fact, adaptive arrays provide an automatic adjustment of the radiation pattern, according to the incident signals and a given adaptation criterion, associated with an efficient signal processing algorithm [1],[2]. Such algorithms are generally based on space-time theory, involving DOA estimation, and impose some a priori knowledge on the number of interference as well as on desired signals.

The adaptive antenna array has been shown as an interesting tool for GPS applications because the system requires increasingly quality and feasibility [1].

In this paper, a hybrid algorithm solution for adaptive antenna array is proposed, which is based on a Frost (FR) [8] and Resende (RES) [6] constrained adaptive algorithm, associated with the MUSIC algorithm for direction of arrival (DOA) estimation, and on a multilayer perceptron neural network (MLPNN).

Two hybrid proposals, using this two kinds of constrained algorithms, are compared to each other and to the classical solutions. Simulations have been carried out considering different realistic GPS situations.

The article is structured as follows. Next section describes the adaptive antenna array and the related signal processing. In the following section, classical solutions and our new hybrid signal processing constrained algorithms are proposed, afterwards simulations results are presented and discussed. Finally, the conclusions summarizes the paper.

ADAPTIVE ANTENNA ARRAY AND THE SIGNAL PROCESSING

Adaptive array theory has undergone extensive developments and has been used in applications linked with radar, geophysics, mobile communications and GPS.

In adaptive spatial filtering, the filter process spatial samples of a wave front captured by an antenna array. For the antenna, the direction of arrival of the incoming signal plays the same role as frequency for the temporal filter. The radiation pattern, which plays for spatial domain the same role as the frequency spectrum for the temporal filter, shows the array sensibility in relation with the direction of arrival of the captured signals.

The concept of spatial filtering is to modify the antenna radiation pattern according to some pre-established criterion, which optimizes reception of the desired signals. The antenna is no longer a passive subsystem acting as a transparent transducer, but an active device which controls the radiation pattern performance based on an intelligent processing.

One way to solve for this problem is to design a multiple antenna, such that elements are physically arranged into an array which can be linear, planar or circular. This array can steer nulls toward interference that provides a control for each element of the array, in a manner that it effectively creates a nearly hemispherical gain pattern when there is no external interference.

This array can detect the presence of interference and to steer a null in its hemispherical gain pattern toward each external interference. The degree of freedom here is limited by the array elements number, generally $M-1$ nulls for M array elements and the depth of the null is limited by the number of nulls that is been steered at the same time. We need to consider that if we have a desired signal and a interference signal coming from the same region, the desired signal could being null together with the interference signal but it is better than have all desired signals suppressed by an interference.

In this context, we will analyze the performance of adaptive linear antenna arrays in the following sections, in order to mitigate the interfering signals by the insertion of nulls in the radiating pattern in the interference directions.

The linear antenna array is uniformly spaced, with M identical isotropic elements and can be visualized in figure 1. Each element is weighted with a complex weight. The mathematical model can be described by:

$$\mathrm{u_n}(k)=\sum_{i=0}^{D+I-1}\mathrm{a_n}(\phi_i)s_i(k)e^{j(f_n(\phi_i))}, n=0,\ldots,M-1 \quad (1)$$

Where: $a_n(\phi_i)$ is the complex response of the n-th array element in direction ϕ_i and $f_n(\phi_i) = -2\pi n \frac{d}{\lambda} sen\phi_i$ is a function associated with the linear geometry.

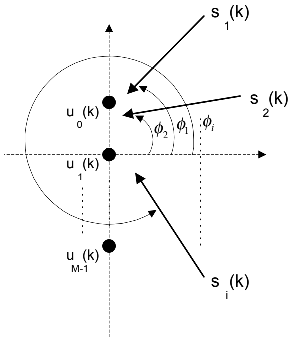

Figure 1: Linear array.

Based on this model, a hybrid algorithm solution is proposed, which switches between a constrained adaptive algorithm (classical approach) and a supervised multilayer perceptron neural network (MLPNN).

SIGNAL PROCESSING ALGORITHMS: CLASSICAL AND HYBRID SOLUTION

The use of signal processing constrained algorithms as a solution for interference mitigation in GPS antenna array is achieved by the imposition of constraints to the antenna weights, in order to assure a constant amplitude and linear phase response in the direction of the desired signal. The constraints are imposed after the estimation of the desired signal direction of arrival (DOA), using the MUSIC algorithm [4].

First, we will describe three classical supervised algorithms: the Linear Constrained Minimum Variance–LCMV proposed by Frost [8]; the Constrained Fast Least-Square – CFLS proposed by Resende et al. [6] and a Multilayer Perceptron Neural Network - MLPNN [10]. After this, we will describe the proposed hybrid algorithm, defined by FR/MLPNN and RES/MLPNN.

FROST ALGORITHM

The so-called Frost (FR) algorithm is a constrained-LMS-based adaptive technique. The Linearly Constrained Minimum Variance (LCMV) criterion consists of minimizing the output power of a filter whose input can be applied in spatial filter, and the parameters are subject to a set of linear constraints. This is done in each iteration in order to move the coefficients vector in the negative direction of the cost function gradient, plus the constraint function obtained by Lagrange multipliers method [12].

The Lagrange multipliers λ are chosen in a way that the weight vector $\mathbf{w}(k)$ have the constraints reached. For this, consider the correlation matrix $\mathbf{R}_{xx}$ knew, so at the *k* instant the gradient function to be minimized is:

$$\nabla w(k) F_{LCMV}[\mathbf{w}(k)] = \mathbf{R_{xx}} \mathbf{w}(k) + \mathbf{C}\lambda(k) \quad (2)$$

After the *k*-th iteration the updating formula of the coefficients vector is:

$$\mathbf{w}(k+1) = \mathbf{w}(k) - \mu[\mathbf{R_{xx}} \mathbf{w}(k) + \mathbf{C}\lambda(k)] \quad (3)$$

where μ is the step size.

The Lagrange multipliers must to be chosen in order to **w(**k+1**)** fulfill the constraints. This set of constraints is imposed by:

$$\mathbf{C^T} \mathbf{w}(k) = \mathbf{f} \quad (4)$$

Where **C** is the constraint matrix and **f** is given by:

$$\mathbf{f} = \lfloor f_{0,} f_1, \ldots, f_{D+I-1} \rfloor, \quad (5)$$

so that f_i is equal to :

0 dB , for $i = 0, \ldots, D-1$, to the desired signals.

-30 dB , for $i = D, \ldots, D+I-1$, (6)

to the interference signals, and

$$\mathbf{C} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ e^{jf(\theta_1)} & e^{jf(\theta_2)} & \cdots & e^{jf(\theta_{D+I-1})} \\ e^{j2f(\theta_1)} & e^{j2f(\theta_2)} & \cdots & e^{jf(\theta_{D+I-1})} \\ \vdots & \vdots & \ddots & \vdots \\ e^{j(M-1)f(\theta_1)} & e^{j(M-1)f(\theta_2)} & \cdots & e^{j(M-1)f(\theta_{D+I-1})} \end{bmatrix} \quad (7)$$

After algebraic manipulations the updating formula is given by:

$$\begin{aligned} \mathbf{w}(k+1) = \mathbf{w}(k) - \mu[\mathbf{I} - \mathbf{C}(\mathbf{C}^T\mathbf{C})^{-1}\mathbf{C}^T]\mathbf{R_{uu}}\mathbf{w}(k) + \\ + \mathbf{C}(\mathbf{C}^T\mathbf{C})^{-1}[\mathbf{f} - \mathbf{C}^T\mathbf{w}(k)] \end{aligned} \quad (8)$$

As observed in [12] the updating equation (8) shows that the factor $[\mathbf{f}-\mathbf{C}^{T}\mathbf{w}(k)]$ is not null, because only if at the *k*-th iteration the coefficients vector reach the constraint accurately this will occur. This fact normally is not achieved because of finite numerical accuracy that can occur in the mathematical operations and values representation.

This fact gives to Frost algorithm the characteristic to numerical robustness to constraint shifts in the adaptation process.

RESENDE ALGORITHM

A constrained fast recursive least square algorithm, know as CFLS [6],[12] can be considered as a least-square version of the Frost technique and can solve problems in constrained adaptive spatial filtering. The CFLS is an algorithm derived directly from the optimum solution, as RLS algorithm. The convergence speed is greater than the algorithms based in gradient, for exemple Frost algorithm.

The complete derivation of Resende (RES) algorithm is too extensive task to be presented here, so a brief will be discussed. In order to take into account the same constraints as in the Frost algorithm, that are imposed in each iteration and objective is to reach the optimal solution by a recursive process, then:

$$\mathbf{w}(k)=\mathbf{\Gamma}(k)\left[\mathbf{C}^{H}\mathbf{\Gamma}(k)\right]^{-1}\mathbf{f} \qquad (9)$$

Where :

$$\mathbf{\Gamma}(k)=\mathbf{R}_{\mathbf{xx}}^{-1}(k)\mathbf{C} \qquad (10)$$

$\mathbf{R}_{\mathbf{xx}}$ is the correlation matrix estimation
And

$$\mathbf{R}_{\mathbf{xx}}(k)=\sum_{i=1}^{k}\alpha^{k-1}\mathbf{u}(i)\mathbf{u}^{H}(i) \qquad (11)$$

The equation (4) can be written as:

$$\mathbf{w}(k+1)=\mathbf{Q}(k+1)\mathbf{f} \qquad (12)$$

where:

$$\mathbf{Q}(k+1)=\mathbf{\Gamma}(k+1)\left[\mathbf{C}^{\mathrm{H}}\mathbf{\Gamma}(K+1)\right]^{-1} \qquad (13)$$

As previously we know that the algorithm can be derived if the recursion is obtained for $\mathbf{Q}(k+1)$ and if this recursion is numerically controlled,

$$\mathbf{x}(k+1)=C^{H}\mathbf{g}(k+1) \qquad (14)$$

and

$$\mathbf{y}^{H}(k+1)=\mathbf{u}^{H}(k+1)\mathbf{Q}(k) \qquad (15)$$

where $\mathbf{u}(k+1)$ is the incoming signal, then the following recursion is obtained:

$$\mathbf{Q}(k+1)=\mathbf{Q}'(K+1)+\mathbf{C}\left[\mathbf{C}^{H}\mathbf{C}\right]^{-1}\left[\mathbf{x}\mathbf{C}^{H}\mathbf{Q}'(K+1)\right] \qquad (16)$$

If $\mathbf{Q}'(k+1)$ denotes a matrix with arithmetical inaccuracy errors, a correcting term can be introduced and the updating matrix is given by:

$$\mathbf{Q}'(k+1)=\left[\mathbf{Q}(k)-\mathbf{g}(k+1)\mathbf{y}^{H}(k+1)\right]$$

$$\left[\mathbf{I}+\frac{\mathbf{x}(k+1)\mathbf{y}^{H}(k+1)}{1-\mathbf{y}^{H}(k+1)\mathbf{x}(k+1)}\right] \qquad (17)$$

where $\mathbf{g}(k+1)$ is the adaptive gain obtained by RLS algorithm, **x** and **y** are auxiliary vectors and **Q'** is an auxiliary matrix.
The initial conditions are:

$$\begin{aligned}\mathbf{w}(0)&=\mathbf{Q}(0)\mathbf{f}\\ \mathbf{Q}(0)&=\mathbf{\Gamma}(0)\left[\mathbf{C}^{H}\mathbf{\Gamma}(0)\right]^{-1}\\ \mathbf{\Gamma}(0)&=\mathbf{R}_{\mathbf{xx}}^{-1}(0)\mathbf{C}\end{aligned} \qquad (18)$$

The constrained algorithm structure used here was developed in parallel, and in this way each adaptive algorithm is associated with a desired signal. Figure 2 shows the constrained algorithm structure.

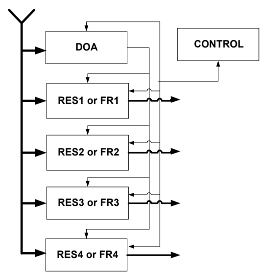

Figure 2: Constrained Algorithm Structure

THE MULTILAYER PERCEPTRON NEURAL NETWORK

The perceptron is the simplest form of a neural network, consisting of a single neuron with adjustable synaptic weight and a threshold [10].

The multilayer perceptron (MLP) is an important class of neural networks called multilayer feedforword networks (MLPNN). Typically, consists of a set of sensor units that are the input layer, one or more hidden layers and output layer of computation nodes. The input signal propagates through the network in a forward direction, on a layer by layer basis.

In the MLP, the model of each neuron includes a non-linearity at the output end. The common form of non-linearity that satisfies this requirement is a sigmoidal defined by:

$$y_j = \frac{1}{1+\exp(-v_j)},\ j = 0,\dots,H-1 \qquad (19)$$

Where v_j is the network internal activity level of neuron *j*, and y_j is the output of the neuron. In our case, we will use a hyperbolic tangent as a sigmoidal non-linearity, which is anti-symmetric with respect to the origin and for which the amplitude of the output lies inside the range $-1 < y_j < 1$. The hyperbolic tangent is defined by:

$$\varphi(v) = a\tanh(bv) \qquad (20)$$

Where *a* and *b* are constants. The hyperbolic tangent is just the logistic function biased and re-scaled, as shown by:

$$a\tanh(bv) = \frac{2a}{1+\exp(bv)} - a \qquad (21)$$

The network contains one or more layers of hidden neurons that are not part of the input or output of the network. These hidden neurons enable the network to learn complex tasks[10].

The network exhibits a high degree of connectivity, determined by the synapses of the network, which determine the MLPNN computational power.

These characteristics are also responsible for the MLPNN drawbacks. The presence of the distributed form of non-linearity and the high connectivity of the network makes the theoretical analysis of multilayer perceptrons difficult to perform and the use of hidden neurons turns problematic the learning process.

The MLPNN is trained by mean of the algorithm, which can be summarized as:

input vector: $\mathbf{x} = [1, x_0, x_1, \dots x_{M-1}]^T$, with *M* input.

output vector: $\mathbf{y} = [y_0, y_1, \dots y_{N-1}]^T$, with *N* input.

Weight matrix between the input layer and hidden layer, for *H* neurons, considering the bias:

$\mathbf{A}_{(M+1)xH}$

Weight matrix between the hidden layer and output layer, for *H* neurons, considering the bias:

$B_{(H+1)xN}$

Considering the hyperbolic tangent as activation function:

$$\mathbf{y} = \left[\frac{e^{\tau\mathbf{x}^T\mathbf{A}} - e^{-\tau\mathbf{x}^T\mathbf{A}}}{e^{\tau\mathbf{x}^T\mathbf{A}} + e^{-\tau\mathbf{x}^T\mathbf{A}}}\right]\mathbf{B} \qquad (22)$$

where, τ control the sigmoid inclination and the MLPNN parameter updating are obtained according to the error function minimization [10].Figure 3 shows the MLPNN structure.

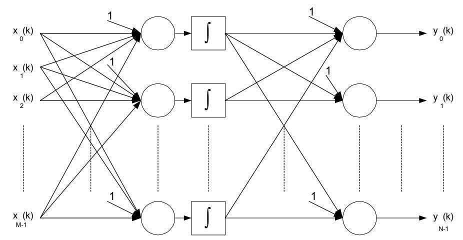

Figure 3 – MLPNN Structure

HYBRID SOLUTION

The hybrid solution has evolved from the requisite to have better accuracy, lower computational burden, lower stead-state error and mitigation of interference, improving in his way the antenna array performance for GPS applications.

The hybrid solution is based on a constrained adaptive algorithm, associated with MUSIC algorithm for direction of arrival (DOA) estimation, and a multilayer perceptron neural network (MLPNN).

The assembly formed by the set DOA estimator, jointly with the constrained adaptive algorithm, that can be Frost or Resende, is used for updating the antenna array. Just with the convergence is reached, the structure is switched to a MLPNN, which receives the constrained adaptive algorithm solution as a training sequence. The adaptation of MLPNN is carried out until a new evaluation of DOA is necessary, due the change of satellite elevation and azimuth angles with the time.

The structure of the algorithm applied here is showed in the block diagram in fig. 4.

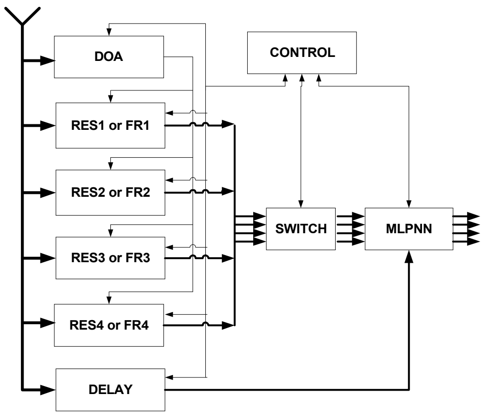

Figure 4 – Hybrid Algorithm Solution

SIMULATIONS RESULTS

Simulations of two different scenarios have been carried out in order to evaluate the hybrid solution algorithm performance.

For the first scenario (sc1), so called critical scenario, it was chosen the desired signals (Des) coming from satellites with elevation of 0°, 60°, 330° and 300°, and the interfering signals (Int) from 315°, 85° and 275°. For the second one (sc2), so called normal scenario, the desired signals comes from elevation angles of 18°, 60°, 80° and 30° and the interfering signals comes from 45°, 300°, 350°, in according to the reference in fig. 1. The desired and interfering angle spectrum estimation was performed by MUSIC algorithm.

The linear antenna array was formed by 8 isotropic elements and were established 7 incoming signals. The same constraints was established for both scenarios, to cancel interfering signals with -30 dB gain and capture the desired signals with 0 dB gain. The signal noise ratio (SNR) and signal interfering ratio (SIR) was established very strong, SNR equal -14.8 dB and SIR equal 3dB. When the MLPNN was used the number of neurons was 30.
The performance of Resende algorithm (RES) for the first scenario (sc1) and Frost algorithm (FR) for the second one (sc2) in the classic solution is showed in fig. 5 to 8.

Observing the temporal evolution (figs. 5 and 7) it is clearly noted the convergence problems take place, mainly when the antenna array is receiving the signals from sc1. The small degree of freedom between the number of the antennas and the incoming signals did not make possible the noise cancellation, consequently did not allowed the open eye condition in two of algorithm output. This is due the high symbol error rate (SER) provided by the constrained algorithm, which is 0.0931.

It is important to consider that in the sc1 the interfering signals were located very close with the desired signals and in angles near to the horizon, factors that made difficult the algorithm convergence. In the sc2, this angle distribution is less critical but the degree of freedom remains small.

The result can be analyzed in the radiation pattern. From figs. 6 and 8, where could be observed a good agreement with the constraints in the capture of all desired signals, as well as the interference cancellation. The FR algorithm in the scenario sc1 shows similar performance. The RES algorithm performance in the scenario sc2 is compared with Frost algorithm in sc2. For this last one a SER decrement to 0.002 is achieved.

The hybrid solution RES/MLPNN and FR/MLPNN are robust. This can be observed in figs. 9 and 11. The comparison between these two solution enables to conclude that the MLPNN achieves the convergence independently the high (sc1) and low (sc1) SER in the training sequence from FR or RES algorithm, with the enough iterations number. While the FR or RES do not achieve that goal.

This can be observed again in the array radiation pattern, figs. 10 and 12, where it is easy to verify that the desired signals are attained according to the constraints requisites, while the interference signal are properly canceled. These structures attain the open eye condition without any problem. The convergence is achieved because of angular space between desired and interfering signals.
For both cases (classical and hybrid), when the degree of freedom between the antenna array elements and incoming signals increase, the SER decrease and a convergence improvement in RES and FR algorithm is reached. For the MLPNN the increase of the degree of freedom allows a decrease of the neurons number and a lower computational burden with respect to the classical solution.

In fig. 13 and 14, it is shown the simulations results, with the same constraints, but for 10 array elements and 20 neurons for the hybrid structure RES/MLPNN in the sc1.

The computational burden of this hybrid solutions have origin in Resende and Frost algorithm associated with the MUSIC algorithm, and it is in order to square number of array elements. The MLPNN has computational complexity in order to $4H(M+5)+3(H+1)N$ sums and $2H(M+4)+2(H+1)N$ multiplication.

CONCLUSIONS

This work points out that spatial processing techniques provide new perspectives in applications related with GPS. The use of hybrid algorithms lead to good solutions where the interfering and multipath signals need to be canceled. Others scenarios, that made a better representation of GPS problem will be established in order to test the hybrid structure.

Different antenna array geometric configuration will be tested, in order to compare the performance, testing less antenna elements and ambiguity resolution. Future studies will work on in the way of have DOA estimators with lower computational burden.

In this work, the computational burden of the hybrid algorithm, is not forbidden in comparison with the reached performance on the task of cancel interfering signals, convergence speed and good results of the array radiation pattern.

ACKNOWLEDGMENTS

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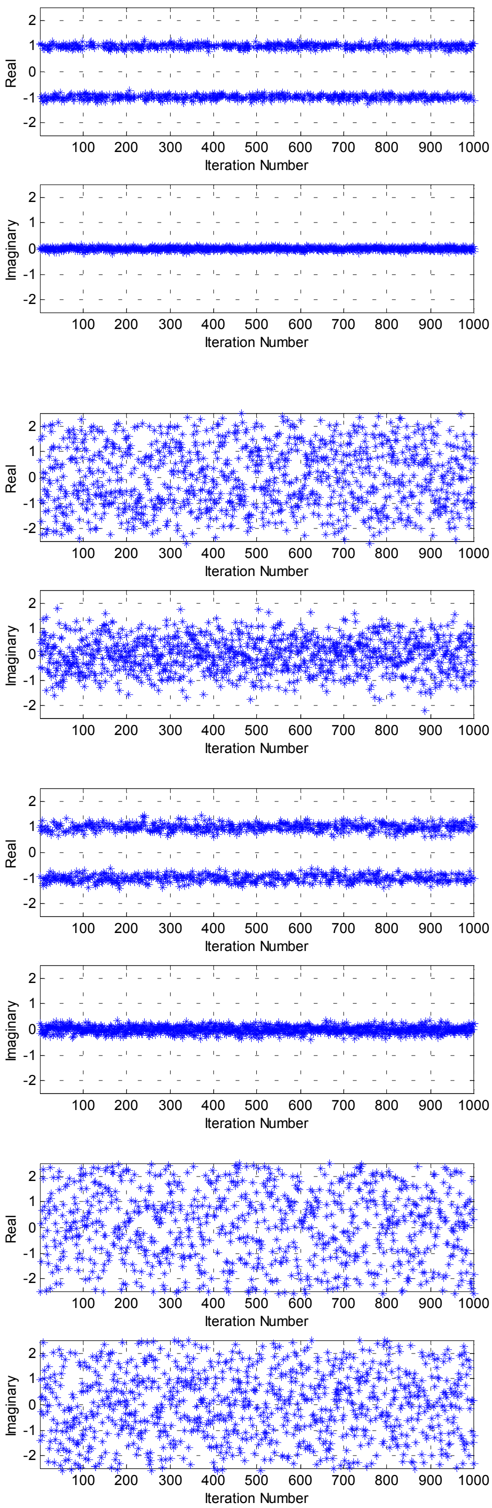

Figure 5 – Temporal Evolution
RES – sc1 - output 1 to 4

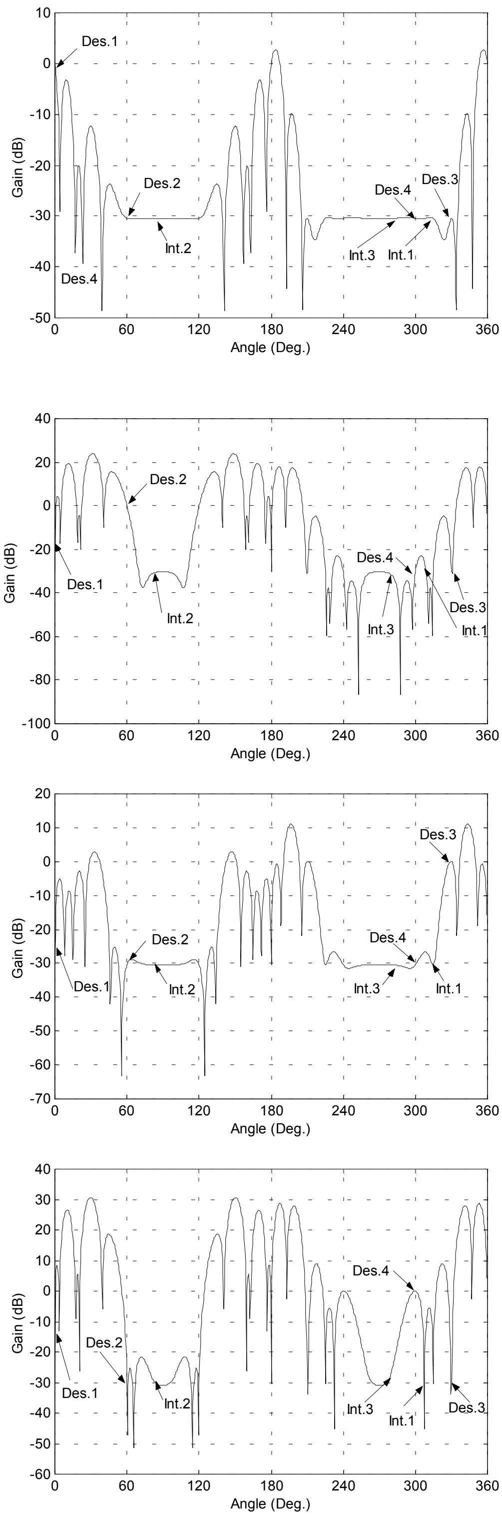

Figure 6 – Array Radiation Pattern
RES- sc1 – output 1 to 4

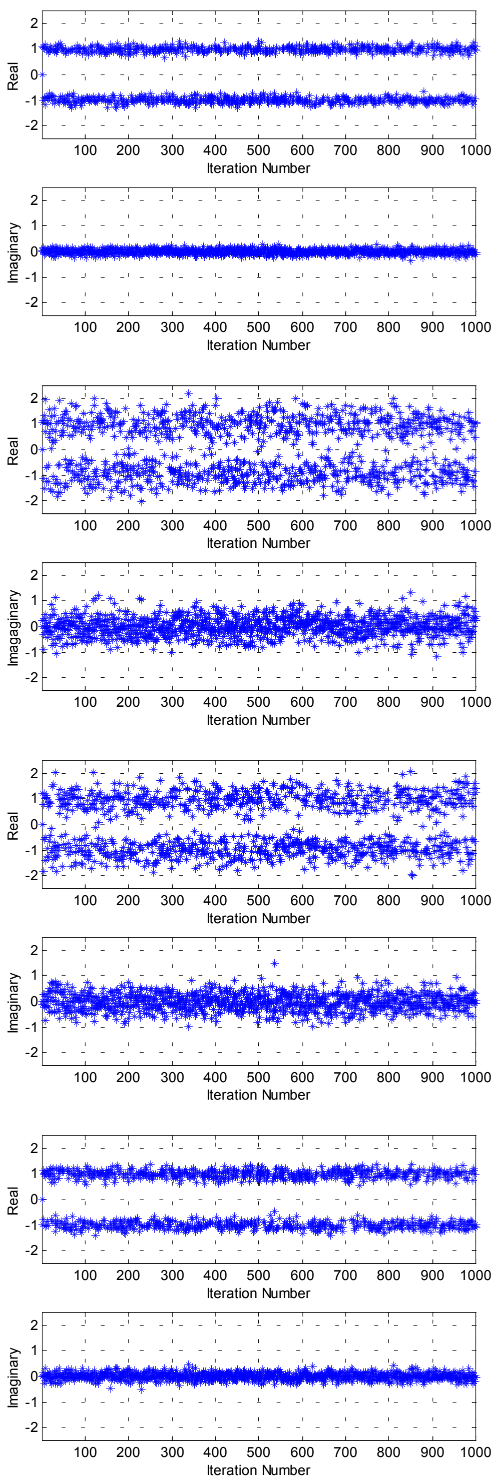

Figure 7 – Temporal Evolution
FR –sc2 – output 1 to 4

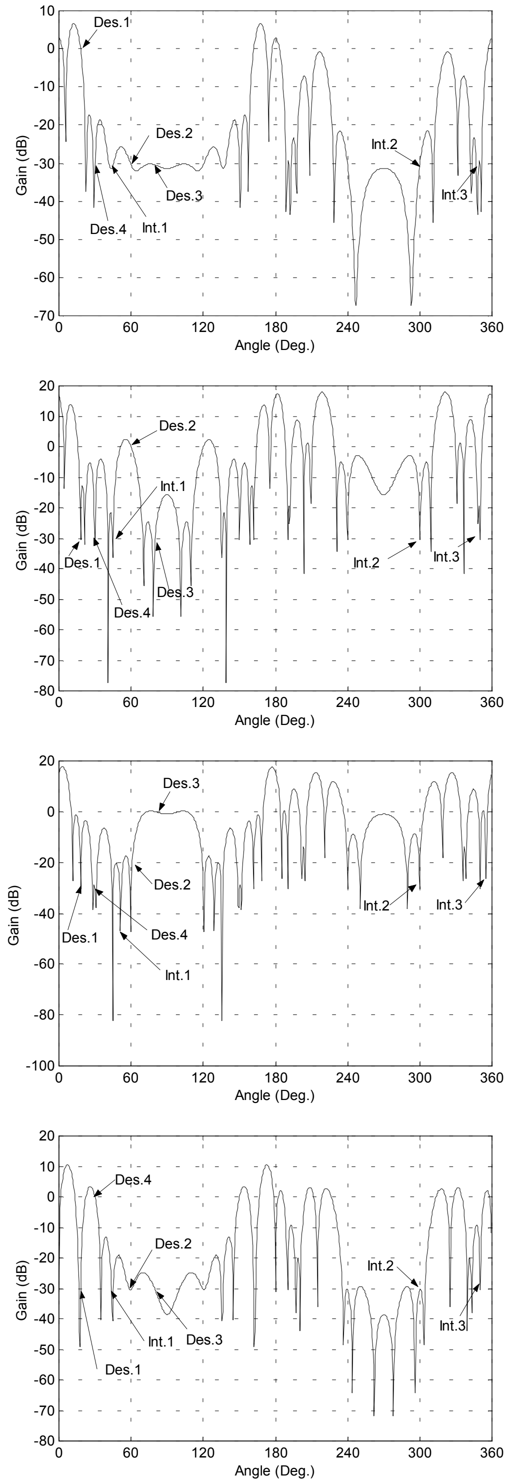

Figure 8 – Array Radiation Pattern
FR – sc2 – output 1 to 4

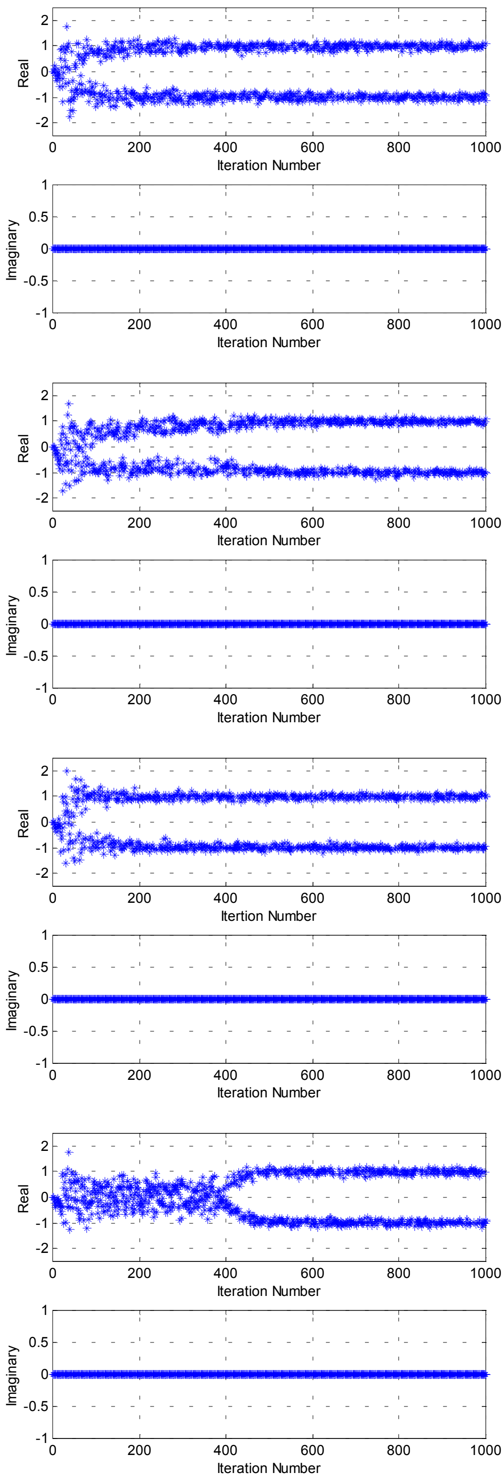

Figure 9 – Temporal Evolution
RES/MLPNN – sc 1 – output 1 to 4

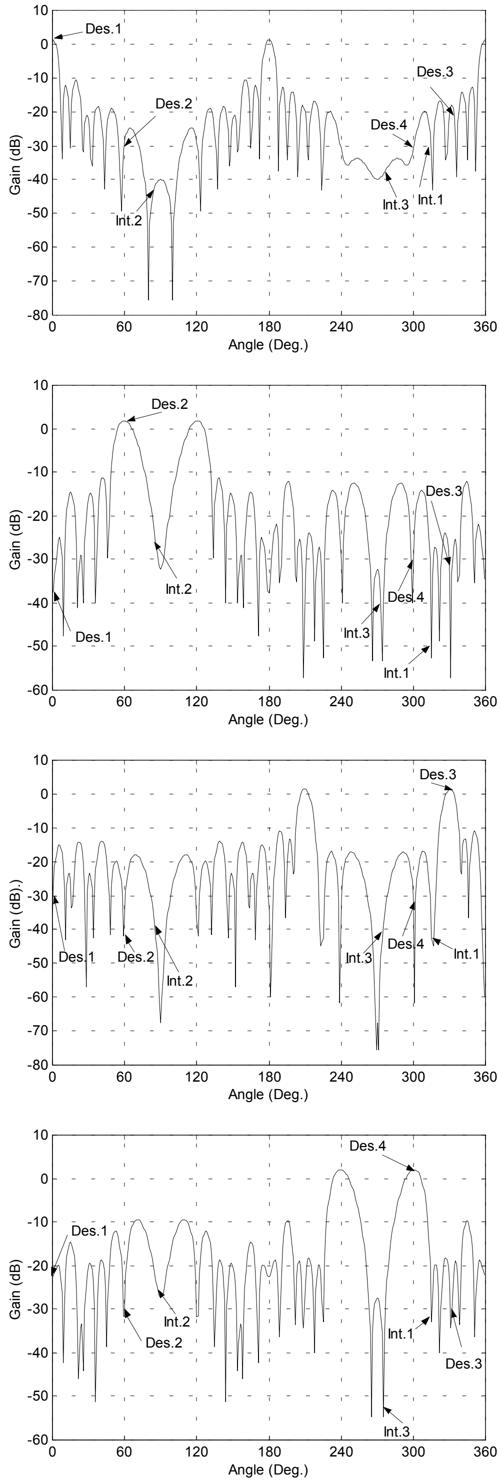

Figure 10 – Array Radiation Pattern
RES/MLPNN – sc1 - output 1 to 4

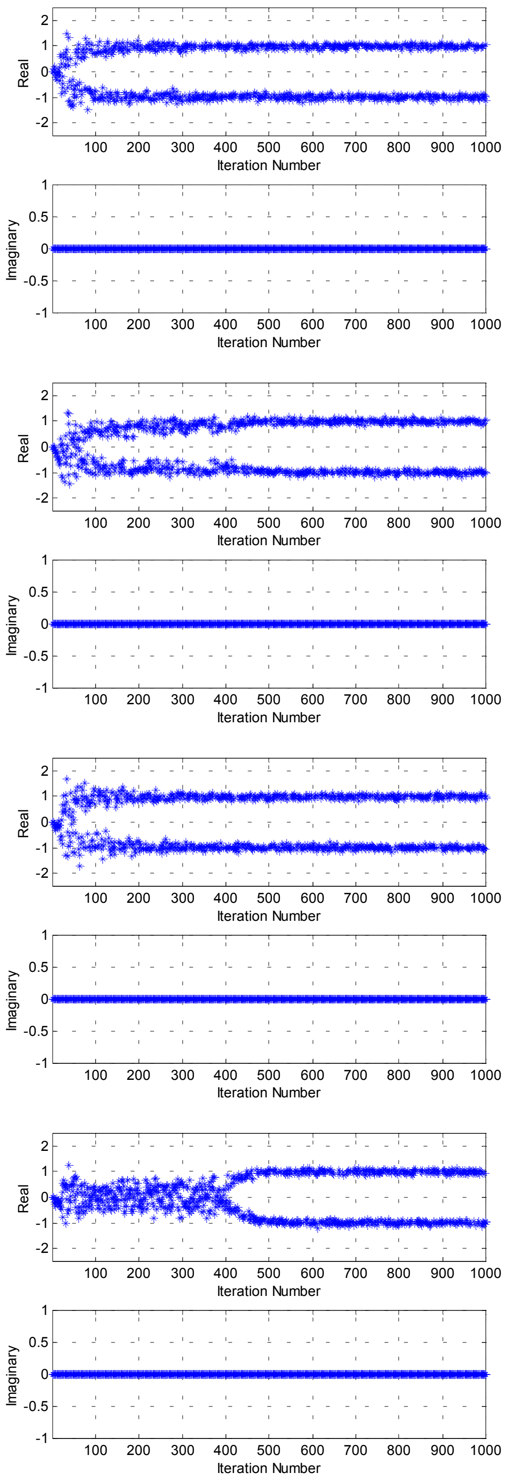

Figure 11 – Temporal Evolution
FR/MLPNN – sc1 – outpu 1 to 4

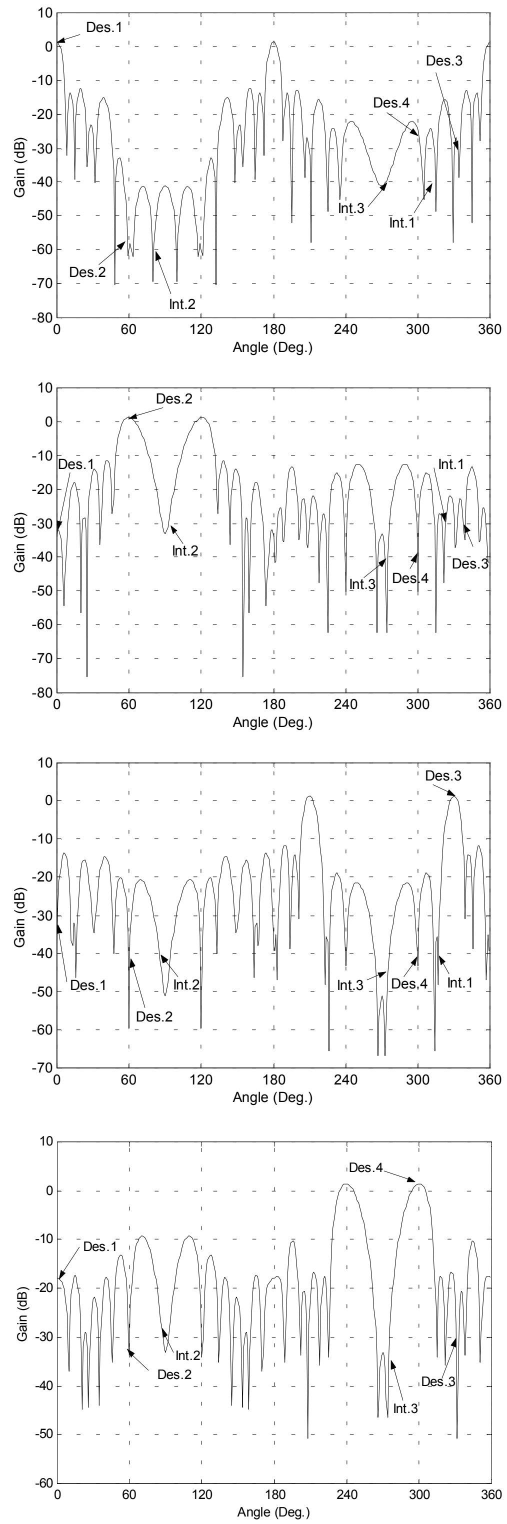

Figure 12 – Array Radiation Pattern
FR/MLPNN – sc1 - output 1 to 4

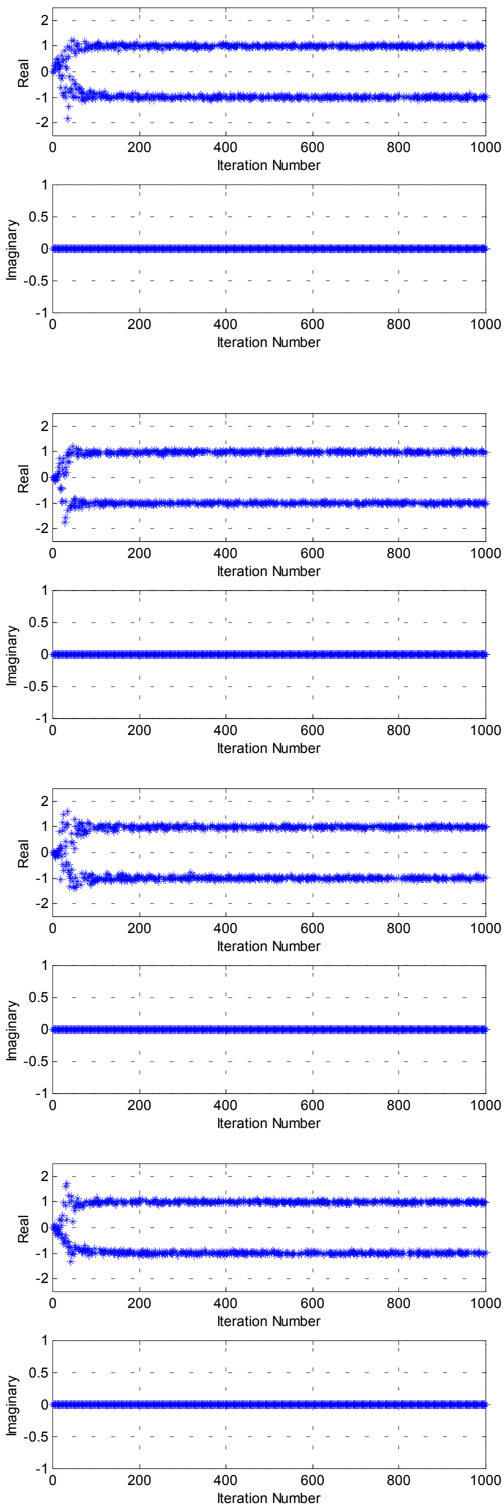

Figure 13 – Temporal evolution
RES/MLPNN – sc1 – 20 neurons – 10 antennas
Output 1 to 4

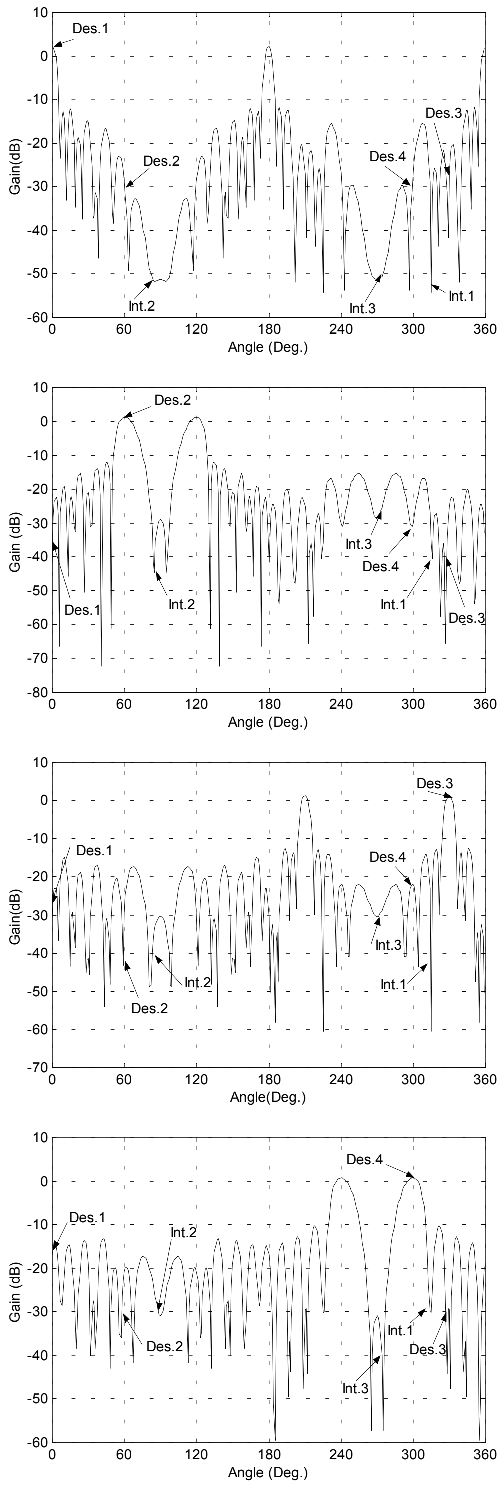

Figure 14 – Array Radiation Pattern
RES/MLPNN – sc 1 – 20 neurons – 10 antennas
Output 1 to 4