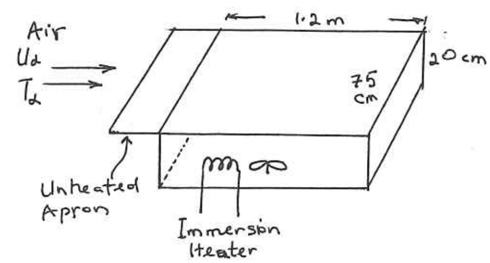
ENCH 501 F2018 **Quiz #5** Name:________________________________

November 20, 2018 Time Allowed: 45 minutes aJ

Very few places on earth with temperate or cold climate have central systems for heating homes. Such systems, under the control of thermistors, turn heat on and off to maintain programmed or pre-set temperatures in typical North American houses heated by natural gas, heavy (or furnace) oil or electricity. In most places, heat is generated by burning wood, coal and from stand-alone devices through which electrical currents are passed. The latter devices are called space heaters and they include direct radiant energy emission from glowing red metal wires and ribbons, with and without fans, and "radiators" filled with oils that are electrically heated. (In relations to income levels, electrical heating is expensive in many countries.) Air flowing over the radiator gains heat and warms up room within which it is circulating. The question is on a primitive model of an oil radiator.

A rectangular metal container, 1.2 m long, 75 cm wide and 20 cm deep, is completely filled with silicone oil. The container is in a large room maintained at a constant temperature of 20°C. An apron is attached to the edge of the top of the container, as per the sketch. The apron, 30 cm by 75 cm, provides an unheated zone in front of the leading edge of the container. The oil is heated to and maintained at 90°C by an immersion coil (electric heater). The oil is well agitated, its temperature is uniform and the same as the temperature of the top surface of the container. All other surfaces of the container are insulated. The free stream velocity of air blown over the exposed top surface, along the long edge, is 1.8 m/s.



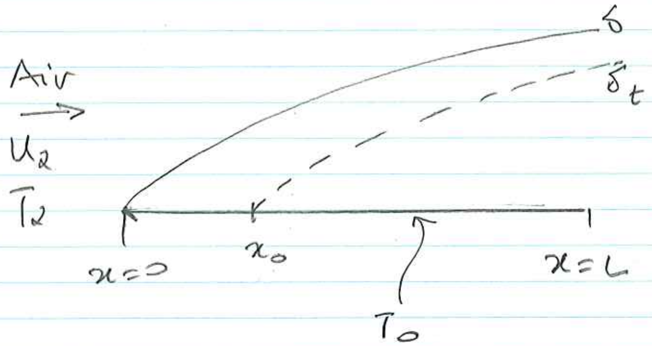
a) <u>Estimate</u> the steady load (in Watts) for the immersion heater. Derivations not necessary but show important steps.

b) For this system, is the assumption involved in the analysis that $\delta_t < \delta$ valid for this system?

Data: Properties of air at 20C - ρ = 1.2047 kg/m^3; μ= 1.817 (10^{-5}) Pa.s; C_p = 1004 J/kg K; k = 0.02563 W/mK; Pr = Cp μ/k.

Hint: An integral may be approximated by a summation over discrete intervals. Values of $\boldsymbol{h_x}$ calculated at different distances $\boldsymbol{x}$ may be used to estimate the integral of $\boldsymbol{h_x}$ over $\boldsymbol{x}$ by the trapezoidal rule. Start the calculation at x = 0.30001, not 0.3.

ENCH 501 Quiz #5 F18 Solution A. Jeje



The problem is to estimate the rate at which energy is transferred from the top surface of the container into air flowing over it.

This amount of energy must be replaced by the immersion heater.

The first step is to establish that the b.l. is laminar. This is true if

$$Re_L = \frac{u_\infty L \rho}{\mu} < 5(10^5) \quad ; \quad L = 1.5\,m$$

$u_\infty = 1.8\ m/s$

$\nu = 1.508\,(10^{-5})\ m^2/s$

$$Re_L = \frac{1.8\,(1.5)}{1.508}(10^5) = 1.79\,(10^5)$$

$\therefore$ b.l. is laminar.

(a) For this part, $x_o = 0.3\,m$

From the Notes,

$$\xi = \frac{\delta_t}{\delta} = \frac{1}{1.026} Pr^{-\frac{1}{3}} \left[1 - \left(\frac{x_o}{x}\right)^{3/4}\right]^{\frac{1}{3}}$$

eq. 6.114

and $$\delta = 4.64\sqrt{\frac{\nu x}{u_\infty}}$$ eq. 6.25

The heat transfer rate from the top surface of container into air is given by

$$Q = \int_{x_o}^{L} h_x (T_o - T_\alpha)\, dx \cdot W$$

$$= (T_o - T_\alpha) W \int_{x_o}^{L} h_x \, dx$$

where $h_x = \dfrac{-k \frac{dT}{dy}\Big|_{y=0}}{T_o - T_\alpha} = \dfrac{3}{2} k \dfrac{1}{\delta_t}$

The integration, with δ_t substituted, is complicated. Hence some values of δ_t and hence h_x are calculated at some x values, and the integral is approximated by a summation.

$$h_x = 0.332 k \, P_r^{\frac{1}{3}} \left[\frac{U_\alpha}{\nu x}\right]^{\frac{1}{2}} \left[1 - \left(\frac{x_o}{x}\right)^{\frac{3}{4}}\right]^{-\frac{1}{3}}$$

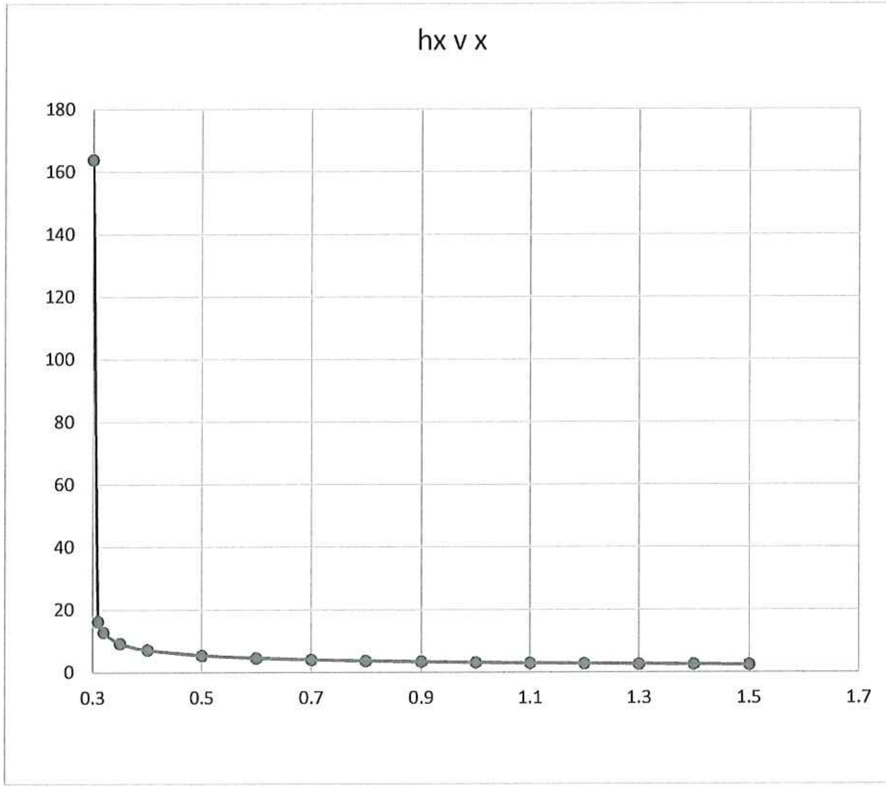
where $k = 0.02563 \; W/mK$, $P_r = \dfrac{C_p \mu}{k} = 0.712$,

$\nu = \mu/\rho = 1.508(10^{-5}) \; m^2/s$, $U_\alpha = 1.8 \; m/s$

and $x_o = 0.3 m$

$$h_x = 2.62515 \left(x^{-\frac{1}{2}}\right) \left[1 - \left(\frac{0.3}{x}\right)^{3/4}\right]^{-\frac{1}{3}}$$

m x	W/m²K hx	Area
0.30001	163.854	
0.31	16.2778	0.899758
0.32	12.83505	0.145564
0.35	9.283396	0.331777
0.4	7.168787	0.411305
0.5	5.43736	0.630307
0.6	4.579016	0.500819
0.7	4.034575	0.43068
0.8	3.647915	0.384125
0.9	3.35447	0.350119
1	3.121729	0.32381
1.1	2.931209	0.302647
1.2	2.771488	0.285135
1.3	2.635064	0.270328
1.4	2.516775	0.257592
1.5	2.412932	0.246485
	Σ	5.77045



$$\therefore \quad Q = (90-20)(0.75)(5.77045)$$

$$= 302.95 \text{ W}$$

In the above, the values of x chosen are important. Smaller steps were chosen near $x = 0.3$. At a uniform Δx throughout, there would be significant errors for $\Delta x > 0.05$, for example.

(b) It is necessary to show that

$$\xi = \frac{\delta_t}{\delta} < 1 \quad \text{at } x = L \text{ for}$$

the system. If this is satisfied, $\xi < 1$ everywhere else.

Hence

$$\xi = \frac{1}{1.024}(0.712)^{-\frac{1}{3}}\left[1 - \left(\frac{0.3}{1.5}\right)^{3/4}\right]^{1/3}$$

$$= 0.9696$$

The assumption holds for the analysis.