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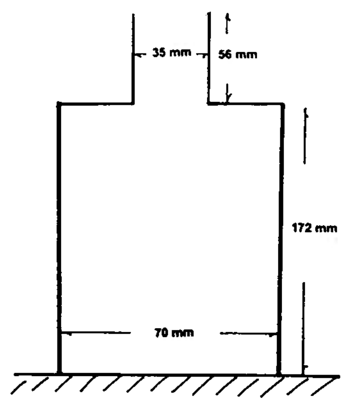
ENCH 501: Transport Processes   Quiz #4   October 10, 2006

Time Allowed: 45 mins.   Name:

Bottles and cans for juices and drinks are returnable for re-use. Such containers must be mechanically washed and sanitized prior to filling. Sanitization disinfects but does not sterilize the containers as some harmless bacteria remain, and these grow in water or drinks to the maximum level in six weeks of unrefrigerated shelf life. Disinfection can be carried out chemically with alkali, chlorine or ozone, or with ultra violet light. With alkali, approved method of sanitization requires that the inside surface of the bottle or metal container be exposed to a minimum 3% by wt. alkali solution (primarily caustic soda, NaOH), and for at least 5 minutes, the bottle is to attain a temperature ≥55°C.

At a bottling plant, soda-lime (or commercial) glass bottles are to be disinfected. The body and the neck of the bottles are shaped as cylinders as per the sketch. External dimensions are given in the diagram. The bottom of the bottle rests on an insulator. The mass of the empty bottle is 308g. When filled to the brim with the alkali solution, the mass is 927g. Alkali at 87.5°C is injected into the bottle at an initial temperature of 18°C and it is assumed that the solution in the bottle is always well mixed. A thermocouple in the bottle (reading in the Celsius scale) recorded that the temperature of the solution decayed exponentially to 61°C in exactly 5 minutes. This decay profile is observed for at least 10 minutes after the solution is introduced and at t = ∞, the solution temperature would have been 0°C (an unrealistic value). The heat released by the solution in part raised the glass temperature. The rest is transferred by convection into the ambient air which is maintained at a constant temperature of 18°C.

If it is assumed that there are no temperature gradients in the wall of the bottle, i.e. it can be treated as a lumped system, and the convective heat transfer coefficient (***h***) external to the bottle is given as 120 W/m^{2}K, is the bottle sanitized 10 minutes after the solution is poured in? Neglect heat losses from the open mouth of the bottle. Show all your derivations.



Data: Glass: Density = 2,530 kg/m^3 ; Specific heat = 0.88 kJ/kg K
Solution: Density = 1,010 kg/m^3; Specific heat = 4.18 kJ/kg K

ENCH 501 Quiz 4 Solution Oct. 10, 2006

(Given:
To sanitize the bottle, it must attain $T \geq 55°C$ for a min. of 5 minutes.) Since the total time given is 10 minutes, the bottle temperature must be raised from 18 °C to 55 °C and kept at or above 55 °C for a duration of 300 s within the period.

+ Perform an energy balance on the bottle.

Input + Gen (→ 0) = Output + Accum.

The input rate is the rate of heat <u>loss</u> by the alkali solution. or

$$\text{input} = -\frac{d}{dt}\left[m_s C_{p_s}(T_s - T_{ref})\right] \quad ; \quad \text{Let } T_{ref} = T_\alpha$$

ambient temp.

T_s = sol'n temp.

$$= -m_s C_{p_s}\frac{dT_s}{dt}$$

$$\text{Generation} = 0$$

$$\text{Output} = hA_o(T - T_\alpha)$$

where A_o is exposed area of bottle

$$\text{Accum} = \frac{d}{dt}\left[m_g C_{p_g}(T - T_{ref})\right] \quad ; \quad T_{ref} = T_\alpha$$

$$= m_g C_{p_g}\frac{dT}{dt}$$

∴

$$-m_s C_{p_s}\frac{dT_s}{dt} = hA_o(T - T_\alpha) + m_g C_{p_g}\frac{dT}{dt} \quad (1)$$

→

It is given that, for $0 \le t \le 600\,s$,

$$T_s = T_{so} \exp(-\beta t) \quad \text{where} \left(\begin{array}{l} T_{so} = 87.5\,^\circ C \text{ and} \\ T_s = 61\,^\circ C \\ \text{at } t = 5\,\text{min or } 300\,s \end{array} \right.$$

i.e. $$\frac{61}{87.5} = \exp(-\beta \, 300)$$

$$\Rightarrow \quad \beta = 0.0012$$

Also $$\frac{dT_s}{dt} = -T_{so}\,\beta \exp(-\beta t) \qquad (2)$$

Substitute (2) into (1)

$$m_s C_{p_s} T_{so}\,\beta\, e^{-\beta t} = h A_o (T - T_a) + m_g C_{p_g} \frac{dT}{dt}$$

Define $\theta = T - T_a$

$$\frac{m_s C_{p_s} T_{so}\,\beta}{m_g C_{p_g}} e^{-\beta t} = \frac{h A_o}{m_g C_{p_g}} \theta + \frac{d\theta}{dt} \qquad (3)$$

or

$$\frac{d\theta}{dt} + P\theta = Q = Q_o e^{-\beta t} \qquad (4)$$

where $P = \dfrac{h A_o}{m_g C_{p_g}}$ and $Q_o = \dfrac{m_s C_{p_s} T_{so}\,\beta}{m_g C_{p_g}}$

Multiply (4) by the integrating factor $e^{\int P dt}$

$$\therefore \quad \theta\, e^{\int P dt} = \int e^{\int P dt}\, Q_o\, e^{-\beta t}\, dt + C$$

$$\theta e^{Pt} = Q_o \int e^{(P-\beta)t}\,dt + C$$

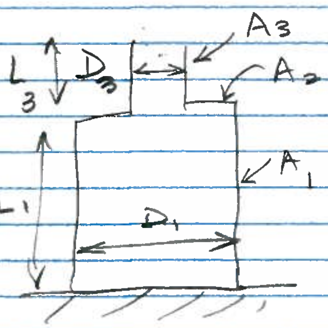
$$= \frac{Q_o e^{(P-\beta)t}}{P-\beta} + C$$

$$\therefore \theta = \frac{Q_o e^{-\beta t}}{P-\beta} + C e^{-Pt}$$

at $t = 0$ $\quad T = T_\alpha \quad \therefore \theta = 0 \quad \therefore C = -\frac{Q_o}{P-\beta}$

$$\therefore \theta = \frac{Q_o}{P-\beta}\left(e^{-\beta t} - e^{-Pt}\right) \qquad (5)$$

Now evaluate parameters.



The exposed area of bottle

$$A_o = A_1 + A_2 + A_3$$

$$= \pi D_1 L_1 + \frac{\pi}{4}\left(D_1^2 - D_3^2\right) + \pi D_3 L_3$$

$$= \pi(0.07)(0.172) + \frac{\pi}{4}\left(0.07^2 - 0.035^2\right) + \pi(0.035)(0.056)$$

$$= 4.7528\,(10^{-2})\ m^2$$

$$P = \frac{h A_o}{m_g C_{p_g}} = \frac{120(4.7528)(10^{-2})}{(0.308)(880)} = 0.021043\ s^{-1}$$

$$Q_o = \frac{m_s C_{p_s} T_{wo} \beta}{m_g C_{p_g}} = \frac{0.619(4180)(87.5)(1.2)(10^{-3})}{0.308\,(880)}$$

$$= 1.00236 \quad K\ s^{-1}$$

$$\theta = T - T_a = T - 18 = \frac{1.00234}{0.021043 - 0.0012}\left(e^{-1.2(10^{-3})t} - e^{-2.1(10^{-2})t}\right)$$

when $t = 300s$, $\theta = 35.15\ °C$

$\therefore\ T = 35.15 + 18 = 53.15\ °C$ →

The bottle at $t = 5$ mins is cooler than the required 55°C. Hence the bottle will not be sanitized in 10 mins from start.

* Note that the transient temperature of the bottle is as in sketch below: