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ENCH 501: Transport Processes Quiz #4 **October 7, 2003**

Time Allowed: 50 mins. **Name:**

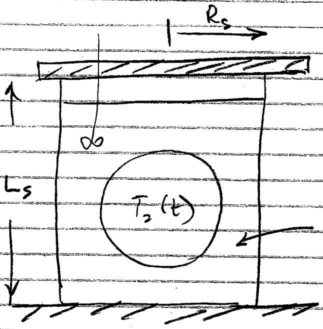
A 6cm diameter copper sphere (ρ = 8920 kg/m^3; c_p = 0.4096 kJ/kg K) is immersed in a bath of oil in a cylindrical metal container. The wall of the container may be considered very thin and therefore it has zero thermal mass. The diameter of the cylinder is 8cm and its height is 7.5cm. (Its temperature is exactly the same as the oil at all times.) The volume of the oil is 250 cm^3, its density is 880 kg/m^3 and its heat capacity is 1.905 kJ/kg K. The container is placed on a mat which is a perfect insulator and the top is covered with a plastic foam board which is also a good insulator. Only the sides of the container are exposed to ambient air at a constant temperature of 18°C. You may assume that the oil is always well mixed and that lumped heat capacity method is valid.

(a) 5 points
If both the oil and the copper were initially at 500°C and the heat transfer coefficient external to the container is given as 68 W/m^2 K, how long will it be before the sphere cools to 45°C? Assume the h around sphere w/ oil is infinite.

(b) 5 points
If the oil was initially at the ambient temperature of 18°C and the copper sphere at 500°C was immersed suddenly, how long will it take the sphere to cool to 45°C if the heat transfer coefficient at the interface between the copper and the oil is given as 42 W/m^2 K while ***h*** for the external surface of the container remains the same as 68 W/m^2 K?

(For part b, full marks will be awarded for setting up the proper expressions and the conditions necessary - i.e. without solving the equations. A bonus of 2 points will be given for the full solution.)

ENCH 501 SOLUTION Quiz #4 Oct. 7, 2003



The ambient temp.

$T_\alpha = 18°C$

For both parts of this problem, the temperatures of the copper sphere and the oil are time-dependent.

Properties.

Sphere – Diam = 6 cm, Volume, $V_2 = \frac{4}{3}\pi R^3 = 113.097(10^{-6})\ m^3$

Surface Area, $A_2 = 113.097\ cm^2 = 113.1(10^{-4})\ m^2$

Density, $\rho_2 = 8920\ kg/m^3$

Heat Capacity, $C_{p_2} = 409.6\ J/kg\,K$

Oil – Volume $V_1 = 250\ cm^3 = 250(10^{-6})\ m^3$

Density, $\rho_1 = 880\ kg/m^3$

Heat capacity, $C_{p_1} = 1905\ J/kg\,K$.

(a) For this part, consider both the sphere and oil as a composite body.

Energy Balance ($T = T_1(t) = T_2(t)$ at all times)

$$-\left(\rho_1 V_1 C_{p_1} + \rho_2 V_2 C_{p_2}\right)\frac{dT}{dt} = h_s A_s (T - T_\alpha)$$

where $h_s = 68\ W/m^2K$ and $A_s = 2\pi R_s L_s$,

$R_s = 4$ cm, $L_s = 7.5$ cm $\therefore A_s = 188.4956(10^{-4})\ m^2$

$T_\alpha = 18°C$

Let $\beta = h_s A_s / \left(\rho_1 V_1 C_{p_1} + \rho_2 V_2 C_{p_2}\right) = 1.54(10^{-3})\ s^{-1}$

The equation is

$$\frac{dT}{dt} = -\beta (T - T_a)$$

or $d \ln (T - T_a) = -\beta \, dt$

Integrate using the conditions

$t = 0 \quad T = 500$

$$\therefore \ln\left(\frac{T - T_a}{500 - T_a}\right) = -\beta t$$

or

$$\frac{T - 18}{500 - 18} = \exp\left[-\beta t\right]$$

When $T = 45°C$, $t = 1871.5$ s or 31.19 mins

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(b) For this part, the sphere transfer heat into the oil which stores part of the heat and transfers the rest to the outside. For this system, both $T_1(t)$ and $T_2(t)$ are changing at different rates, and $T_1 \leq T_2$.

Perform energy balance on sphere as control volume

$$-\rho_2 V_2 c_{p_2} \frac{dT_2}{dt} = h_i A_i (T_2 - T_1)$$

where $h_i = 42 \; W/m^2K$ and $A_i = A_2 = 113.1(10^{-4}) \, m^2$

Let $$\gamma = \frac{h_i A_i}{\rho_2 V_2 c_{p_2}}$$

∴ Equation is

$$-\frac{dT_2}{dt} = \delta\,(T_2 - T_1) \qquad \text{(eq. 1)} \rightarrow$$

This equation has 2 unknowns, thus we need another expression.

Do an overall balance on the system (energy balance on oil)

Rate of heat loss by sphere = Rate of heat gain by oil + Rate of heat loss to ambient

i.e.

$$-\rho_2 \forall_2 c_{p_2} \frac{dT_2}{dt} = \rho_1 \forall_1 c_{p_1} \frac{dT_1}{dt} + h_s A_s (T_1 - T_\alpha)$$

or

$$-\frac{dT_2}{dt_1} - \gamma \frac{dT_1}{dt} = \varepsilon\,(T_1 - T_\alpha) \qquad \text{(eq. 2)} \rightarrow$$

where $\gamma = \dfrac{\rho_1 \forall_1 c_{p_1}}{\rho_2 \forall_2 c_{p_2}}$ and $\varepsilon = \dfrac{h_s A_s}{\rho_2 \forall_2 c_{p_2}}$

Equations (1) and (2) can be combined to eliminate T_1 as follows.

$$-\frac{dT_2}{dt} - \gamma \frac{dT_1}{dt} = \varepsilon\,(T_1 - T_\alpha) \qquad \times\,\delta$$

$$-\frac{dT_2}{dt} = \delta\,(T_2 - T_1) \qquad \times\,\varepsilon$$

Add

$$-(\delta + \varepsilon)\frac{dT_2}{dt} - \gamma\delta \frac{dT_1}{dt} = \delta\varepsilon\,(T_2 - T_\alpha) \qquad \text{(eq. 3)} \rightarrow$$

To eliminate $\frac{dT_1}{dt}$ from eq. 3, differentiate eq. (1).

$$-\frac{d^2T_2}{dt^2} = \gamma\left(\frac{dT_2}{dt} - \frac{dT_1}{dt}\right)$$

where $$\frac{dT_1}{dt} = \frac{1}{\gamma}\frac{d^2T_2}{dt^2} + \frac{dT_2}{dt} \qquad \text{(eq. 4)}$$

Substitute (4) into (3)

$$-(\delta+\varepsilon)\frac{dT_2}{dt} - \gamma\delta\left[\frac{1}{\gamma}\frac{d^2T_2}{dt^2} + \frac{dT_2}{dt}\right] = \delta\varepsilon(T_2 - T_a)$$

or $$+\gamma\frac{d^2T_2}{dt^2} + (\delta+\varepsilon+\gamma\delta)\frac{dT_2}{dt} + \delta\varepsilon(T_2 - T_a) = 0$$

This is a second-order ODE.

$$\frac{d^2T_2}{dt^2} + \frac{(\delta+\varepsilon+\gamma\delta)}{\gamma}\frac{dT_2}{dt} + \frac{\delta\varepsilon}{\gamma}(T_2 - T_a) = 0$$

(eq. 5)

with conditions

$t = 0 \qquad T_2 = T_{20} = 500°C$

$t = 0 \qquad T_1 = T_a = 18°C$ (T_1 related to T_2 via eq. 1)

(also at $t = \infty$, $T_2 = T_a = 18°C$)

We write eq. (5) as

$$\frac{d^2\theta}{dt^2} + a\frac{d\theta}{dt} + b\theta = 0$$

$(D^2 + aD + b)\theta = 0$ in terms of operator

$(D + r_1)(D + r_2)\theta = 0$

where $r_1 r_2 = b$ and $r_1 + r_2 = a$

The solution is $\theta = C_1 e^{-r_1 t} + C_2 e^{-r_2 t}$ (Eq. 6)

Evaluate the constants $\delta, \varepsilon, \gamma, a, b, r_1$ and r_2.

$$\delta = \frac{h_i A_i}{\rho_2 \forall_2 c_{p_2}} = \frac{42(113.1)(10^{-4})}{(8920)(113.1)(10^{-6})\,409.6} = 1.1495(10^{-3})\ s^{-1}$$

$$\varepsilon = \frac{h_s A_s}{\rho_2 \forall_2 c_{p_2}} = \frac{68(188.5)(10^{-4})}{(8920)(113.1)(10^{-6})(409.6)} = 3.1019(10^{-3})$$

$$\gamma = \frac{\rho_1 \forall_1 c_{p_1}}{\rho_2 \forall_2 c_{p_2}} = \frac{880(250)(10^{-6})(1905)}{8920(113.1)(10^{-6})(409.6)} = 1.0142$$

$$a = \frac{\delta(1+\gamma)+\varepsilon}{\gamma} = 5.3415(10^{-3})$$

$$b = \frac{\delta\varepsilon}{\gamma} = 3.5157(10^{-6})$$

$r_2 = a - r_1$

$r_1(a - r_1) = b$

$r_1^2 - ar_1 + b = 0$

Solve.

$$r_1 = \frac{5.3415(10^{-3}) \pm 3.8038(10^{-3})}{2}$$

i.e. $7.6885(10^{-4})$ or $4.5727(10^{-3})$

and $r_2 = a - r_1 = 4.5727(10^{-3})$ or $7.688(10^{-4})$

∴ Solution

$$\theta = T_2 - T_\alpha = C_1 e^{-4.5727(10^{-3})t} + C_2 e^{-7.6885(10^{-4})t}$$

at $t = 0$, $T_2 = 500\,^\circ C$ and $T_\alpha = 18\,^\circ C$

$$482 = C_1 + C_2 \qquad \text{(Eq. 7)}$$

Using equation (6) with (eq. 1)

$$C_1 r_1 e^{-r_1 t} + C_2 r_2 e^{-r_2 t} = \sigma \left\{ C_1 e^{-r_1 t} + C_2 e^{-r_2 t} - (T_1 - T_\alpha) \right\}$$

at $t = 0$ $T_1 = T_\alpha$

$$\therefore \quad C_1 r_1 + C_2 r_2 = \sigma \{ C_1 + C_2 \} \qquad \text{(eq. 8)}$$

$$C_1 (r_1 - \sigma) = -C_2 (r_2 - \sigma)$$

$$C_1 = 0.1112\, C_2 \quad \text{or} \quad C_2 = 8.993\, C_1$$

use this in eq. 7

$$\left\{ \begin{array}{l} C_1 = 482 / 9.993 = 48.2336 \\ r_1 = 4.5727(10^{-3}) \end{array} \right.$$

$\therefore$ Final solution

$$T_2 - 18 = 48.2336\, e^{-4.5727(10^{-3})t} + 433.7664\, e^{-7.6885(10^{-4})t}$$

when $T_2 = 45°C$, $t = 3611.5s$ or 60.19 min

If however $r_1 = 7.6885(10^{-4})$

$$C_1 / C_2 = 8.9924 \quad \Rightarrow \quad C_1 = 433.7664$$

obtain same answer.