# The University of Calgary
## Department of Chemical & Petroleum Engineering

**ENCH 501: Transport Processes Quiz #1** **September 19, 2006**

**Time Allowed: 45 mins.** **Name:**

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**1. (2 points)** Fifteen (15) friends decide to choose a common beverage because they can get a deal as a group. Their preferences are as follows:

- 6 prefer milk > wine > beer
- 5 prefer beer > wine > milk
- 4 prefer wine > beer > milk

If, after the choices had been made, one member of the group received a phone call that wine is unavailable, what beverage should be bought? Explain your reasoning. (Instead of beverages, the issue could have been decisions by a group on purchasing an equipment manufactured by different companies.)

**2. (8 points)** Centrifugal pumps are used extensively in the chemical and petroleum industries. A pilot flow loop is to be scaled-up for a natural gas distribution network. The pumps in the flow loop are thus to be scaled up such that the performances of the pilot and full-scale units are similar. It is given that the hydrostatic head (h, m) a pump develops depend on the volume flow rate (Q, $m^3/s$), the density (ρ, $kg/m^3$) and the viscosity (μ, Pa.s) of the fluid, the diameter (D, m) and the angular speed (ω, rad/s) of the impeller and the power input (rate of work, $P=\rho Qhg$, J/s) into the pump.

a) Determine the dimensionless groups for the pump.
b) The pilot pump has a 20 cm diameter impeller which is rotating at an angular speed of 1170 rpm and it discharges <u>a fluid</u> at a rate of 1134 litres per minutes with a head of 6.675m. What would the *flow rate* and the *head* be, and what *power* is required to drive a 50cm impeller diameter pump <u>for water</u> operated at an angular speed of 1750 rpm? The density of water is 1000 $kg/m^3$. It is assumed that the pumps are 'similar' in all respects - dynamic, kinematic, geometric etc. Is the assumption valid? Can the fluid through the pilot pump be the same water as flows through the large pump?

ENCH 501 Quiz 1 Solution Sept. 19, 2006

1. In terms of preferences, the largest number (6) chose milk. With the added information about unavailability of wine, 9 would now prefer beer to milk versus the original 6 who prefer milk. A paradox has developed. The beverage bought should still be milk because the added intelligence has not change the preferences for the 3 beverages.

2. Given

$$h = f(Q,\ \rho,\ \mu,\ D,\ \omega,\ P)$$

units: $h$: m; $Q$: $\frac{m^3}{s}$; $\rho$: $\frac{kg}{m^3}$; $\mu$: Pa.s $= \frac{N}{m^2}s = \frac{kg}{m^3}\frac{m}{s^2}s$; $D$: m; $\omega$: rad/s $= \frac{1}{s}$; $P$: $\frac{J}{s} = \frac{N \cdot m}{s} = \frac{kg}{s}\frac{m \cdot m}{s^2}$

Dimensions: $h$: $L$; $Q$: $\frac{L^3}{t}$; $\rho$: $\frac{M}{L^3}$; $\mu$: $\frac{M}{Lt}$; $D$: $L$; $\omega$: $\frac{1}{t}$; $P$: $\frac{ML^2}{t^3}$

For the problem, there are 3 fundamental dimensions, M, L and t and there are 7 variables. Thus there will be 4 dimensionless groups formed.

By inspection, $\pi_1 = \frac{h}{D}$

Drop one of the variables, hence

$$P = g(Q, \rho, \mu, D, \omega)$$

form the other $\Pi$s — choose 3 recurring variables — $D, \rho, \omega$ (e.g.)

$$\Pi_2 = D^a \rho^b \omega^c Q$$

$$\Pi_3 = D^a \rho^b \omega^c \mu$$

$$\Pi_4 = D^a \rho^b \omega^c P$$

Apply the Buckingham Pi Theorem to determine the coefficients.

$$\Pi_2 = L^a \left(\frac{M}{L^3}\right)^b \left(\frac{1}{t}\right)^c \frac{L^3}{t}$$ + gp coefficients

M $0 = b$

L $0 = a - 3b + 3$

t $0 = -c - 1$

$a = -3$, $b = 0$, $c = -1$

$$\therefore \quad \Pi_2 = \frac{Q}{D^3 \omega}$$

$$\Pi_3 = L^a \left[\frac{M}{L^3}\right]^b \left[\frac{1}{t}\right]^c \frac{M}{Lt}$$

M $0 = b + 1$

L $0 = a - 3b - 1$

t $0 = -c - 1$

$a = -2$, $b = -1$, $c = -1$

$$\pi_3 = \frac{\mu}{D^2 \rho \omega}$$

and $$\pi_4 = L^a \left[\frac{M}{L^3}\right]^b \left[\frac{1}{t}\right]^c \frac{ML^2}{t^3}$$

$$\begin{aligned} M \quad & 0 = b + 1 \\ L \quad & 0 = a - 3b + 2 \\ t \quad & 0 = -c - 3 \end{aligned} \quad \Bigg\} \quad \begin{aligned} a &= -5 \\ b &= -1 \\ c &= -3 \end{aligned}$$

$$\pi_4 = \frac{P}{D^5 \rho \omega^3}$$

a/ The dimensionless group are hence

$$\frac{P}{D^5 \rho \omega^3} = f\left(\frac{h}{D}, \frac{Q}{D^3 \omega}, \frac{\mu}{D^2 \rho \omega}\right)$$ and the inverses or products.

b/ Given $D_1 = 0.2\,m$

$\omega_1 = 1170\ rpm$

$Q_1 = 1.134\ m^3/min$

$h_1 = 6.675\ m$

for similarity, all the dimensionless gps have to be the same for both pumps.

Assume that the pumps are similar in all respects.

Given $D_2 = 0.5\,m$

then $\frac{h_1}{D_1} = \frac{h_2}{D_2} \Rightarrow h_2 = \frac{D_2}{D_1} h_1 = \frac{0.5}{0.2}(6.475)$

$\therefore \quad h_2 = 16.688\,m$ — head for large pump. ⟶

and

$$\frac{Q_1}{D_1^3 \omega_1} = \frac{Q_2}{D_2^3 \omega_2} \Rightarrow Q_2 = \left(\frac{D_2}{D_1}\right)^3 \frac{\omega_2}{\omega_1} Q_1$$

$$Q_2 = (2.5)^3 \left(\frac{1750}{1170}\right)(1.134) = 26.502\ m^3/min$$ ⟶

The power into large pump is calculated from

$$P_2 = \rho_2 Q_2 h_2 g = 1000\left(\frac{26.502}{60}\right)(16.688)(9.81)$$

$$= 72{,}310.4\ J \quad (\text{or } 72.31\ kJ)$$ ⟶

For complete similarity, we must also have

$$\frac{\mathcal{P}_1}{D_1^5 \rho_1 \omega_1^3} = \frac{\mathcal{P}_2}{D_2^5 \rho_2 \omega_2^3} \quad \text{and} \quad \frac{D_1^2 \rho_1 \omega_1}{\mu_1} = \frac{D_2^2 \rho_2 \omega_2}{\mu_2}$$

or, by definition

$$\frac{\rho_1 Q_1 h_1 g}{D_1^5 \rho_1 \omega_1^3} = \frac{\rho_2 Q_2 h_2 g}{D_2^5 \rho_2 \omega_2^3} \quad \text{and} \quad \frac{\rho_1/\mu_1}{\rho_2/\mu_2} = \frac{D_2^2 \omega_2}{D_1^2 \omega_1}$$

Expand

$$\frac{Q_1}{D_1^3 \omega_1} \cdot \frac{h_1}{D_1} \cdot \frac{g}{D_1 \omega_1^2} = \frac{Q_2}{D_2^3 \omega_2} \cdot \frac{h_2}{D_2} \cdot \frac{g}{D_2 \omega_2^2}$$

$$\Rightarrow \quad D_1 \omega_1^2 = D_2 \omega_2^2$$

This is the ratio the fluid properties must have. The fluid thro the pilot pump is not same water since r.h.s > 1. ⟶

With the data given, this equation cannot be satisfied. That is

$$(0.2)\left(\frac{1170}{60}\right)^2 \neq (0.5)\left(\frac{1750}{60}\right)^2$$

Thus complete similarity cannot be achieved! ⟶