GJ

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ENCH 501: Transport Phenomena

Mid-Term Examination, Fall 2011

Instructions: Time: 8.20 to 9.50 am Oct 17, 2011
Attempt All Questions. Open Notes & Book. Use of calculators permitted.

Problem #1 (10 points)

A vertical cylindrical tank is used to separate small water droplets dispersed in oil by settling. Ordinarily, a fixed amount of the oil (density equal 982 kg/m^3 and dynamic viscosity of 14.6 mPa.s) is charged into the tank (3 m tall and 2 m inside diameter) and the water droplets with dissolved salts (maximum diameter of 1 mm, density equal 1019 kg/m^3) are allowed to settle and separate.
In one cycle, when there are no flows in or out of the tank, much of the water had separated from the oil in the tank. The percentage of water (not settled - as droplets) by volume in the oil was 4%. The water at the bottom of the tank was drained out and the height of oil left was 1.2 m. At this instant, both valves in the supply line (of the feed oil) and the exit pipe failed. Feed oil that contains 10% by volume of water at 15.6°C (as droplets) flowed into the tank at a constant rate of 7.2(10^{-3}) m^3/s, and oil flowed out of the horizontal outlet line, 2cm inside diameter and 1.5 m long, located at the very bottom of the tank. The flow out of the tank is controlled by gravity since the tank is vented to the atmosphere through a large hole at the top. Assume that the tank content is always well mixed and the flow through the exit pipe is always laminar. Show all important steps.

a) After how long will the tank be full of liquid? Assume the average density of liquid in the tank is constant.
b) How much salt (in kg) is present in the tank at this instant? State your assumptions.

Data: At 15.6°C

Mass % salt in water	Density of the solution (kg/m^3)
0	1000
1.056	1007
2.112	1015
4.223	1030
5.807	1042
10.030	1074
20.060	1152
26.395	1204

Problem #2 (15 points)

The following data on the pipes for a drilling string are obtained from the Halliburton "Red Book", 1995.

OD, mm	ID,mm	Mass/length (m_+, kg/m)	Length suspended in a well, L in m	Stretch due to own mass suspended in water, ℓ in mm
114.3	97.18	24.7	1000	150.3

i) Estimate the Young's modulus of the metal for the pipe. Assume the density of water is 1000 kg/m^3, and the stretch (or elongation) is elastic. The acceleration of gravity is 9.81 m/s^2.

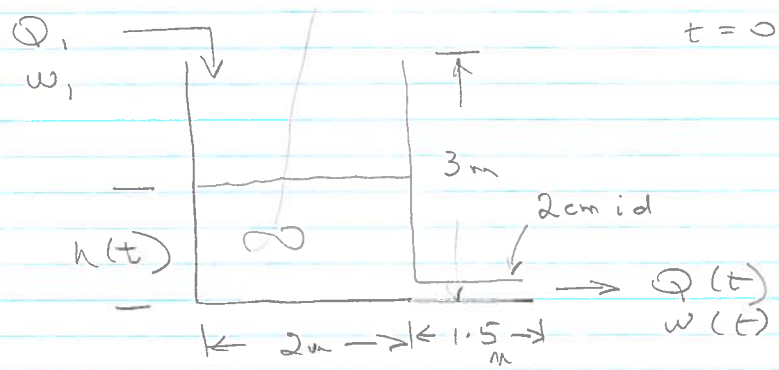
ii) If a pipe string of the same material and properties (as in the table above) is to be used to drill a hole 2000 m deep, estimate the stretch due to its own mass when suspended in water in a well 2 km deep.

iii) With reference to part (ii) above, at what location along the string is the maximum (local) strain? If the limiting strain before the pipe fails catastrophically (breaks suddenly) is $5.6(10^{-4})$, will the string fail if an attempt is made to pull it up at the drilling platform?

Hint: Obtain an expression for the local strain due to the load below the point. Set the lower end of the string at z = 0 and let z increase upwards. The length of the string before stretching is L. Choose a differential element dz at z. The element dz will stretch by $d\ell$ locally under the load below it.

ENCH 501 - Mid Term Solution - Oct. 17, 2011

Q #1



$t = 0$ $h = 1.2$ m
$w = 0.04$

$w_1 = 10\%$ or 0.1
$Q_1 = 7.2\,(10^{-3})\ m^3/s$

(a) The densities of the oil and water are different but the volume of the water is relatively small. Thus, assume that the average density of the liquid in the tank is essentially constant for this part. (With 10% water, the liquid density is 985.7 kg/m³, only marginally greater that 982 kg/m³ for the oil).

For a mass balance on liquid in the tank:

$$\rho_f Q_1 = \rho_t Q + \rho_t A \frac{dh}{dt} \quad ; \quad \rho_f \simeq \rho_t$$

The flow out of the tank, Q, is described by the Hagen-Poiseuille equation

$$Q = \frac{\pi (P_0 - P_1) R^4}{8 \mu L} \quad ; \quad P_0 - P_1 = h \rho g$$

$$\therefore \quad Q = \left(\frac{\pi R^4 \rho g}{8 \mu L}\right) h = \beta h$$

$$\therefore \quad Q_1 = \beta h + A \frac{dh}{dt}$$

$$\text{or} \quad \frac{dh}{dt} = \frac{1}{A}\left(Q_1 - \beta h\right)$$

Let $Y = Q_1 - \beta h$; $dY = -\beta\, dh$

$$\therefore \quad -\frac{1}{\beta}\frac{dY}{dt} = \frac{1}{A} Y \qquad \text{or} \qquad \int_{Y_0}^{Y} \frac{dY}{Y} = -\frac{\beta}{A}\int_0^t dt$$

solve

$$\ln\left(\frac{Y}{Y_0}\right) = -\frac{\beta}{A} t \qquad \text{or} \qquad Y = Y_0 e^{-(\beta/A)t}$$

$$Q_1 - \beta h = \left(Q_1 - 1.2\beta\right) e^{-(\beta/A)t}$$

When the tank fills up, $h = 3$ m

$$\beta = \frac{(3.142)(982)(9.81)(10^{-8})}{8(14.6)(10^{-3})(1.5)} = 1.7274(10^{-3}) \frac{m^2}{s}$$

$$Q_1 = 7.2(10^{-3})\ m^3/s \quad ; \quad A = \pi\ m^2$$

$$\ln\left[\frac{7.2(10^{-3}) - 3(1.7274)(10^{-3})}{7.2(10^{-3}) - 1.2(1.7274)(10^{-3})}\right] = -\frac{1.7274(10^{-3})}{\pi} t$$

$$\therefore \quad t = 1696 \text{ s} \quad \text{or} \quad 28.267 \text{ mins}$$

$\longrightarrow$

(b) Mass balance on the salt in the tank.

$$\rho_f Q_1 w_1 x = \rho_t Q w x + \rho_t A \frac{d(hwx)}{dt}$$

where w is the fraction that is water of the liquid, and x is the mass % of salt in the water. x is constant and is obtained from the table provided by linear interpolation:

$\rho_{sol^n} = 1019 \ kg/m^3$, $x = 2.112 + \frac{4}{15}(2.111)$ %

$x = 2.6749$ % or 0.026749

As before, $Q = \beta h$

$$Q_1 w_1 x = \beta x (hw) + A x \frac{d(hw)}{dt}$$

Like for part (a), solve

$$\ln \frac{Q_1 w_1 x - \beta x (hw)}{Q_1 w_1 x - \beta x (hw)_{t=0}} = -\frac{\beta}{A} t$$

where $h = 3m$, w ? at t

$t = 0$, $h = 1.2m$, $w = 0.04$ and $w_1 = 0.1$

$$\ln \left[\frac{7.2(10^{-3})(0.1)(0.026749) - 1.7274(10^{-3})(0.026749)\,3\,w}{7.2(10^{-3})(0.1)(0.026749) - 1.7274(10^{-3})(0.026749)\,1.2(0.04)} \right.$$

$$= -\frac{1.7274(10^{-3})}{\pi}(1626) = -0.3936$$

Solve $w = 0.0906$ when the tank is full.

Hence amount of salt in the tank =

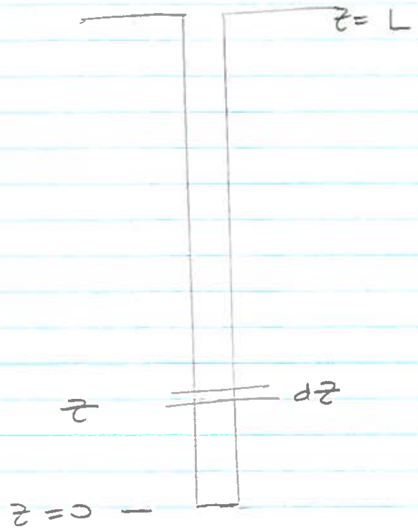
Density × Volume × solution fraction × salt fraction

$$= [982(0.9094) + 1019(0.0906)](\pi 3) \times (0.0906)(0.026749)$$

$$\frac{kg}{m^3} \cdot m^3 \cdot \frac{vol.\ water}{Total\ volume} \times \frac{mass\ salt}{mass\ solution}.$$

Mass of salt $= 22.505$ kgs →

Q # 2.



The element dz is stretched by the load underneath, i.e.

$$\frac{m g z}{A} = E \cdot \frac{dl}{dz}$$

stress   Young's Modulus   local strain

or $$\frac{dl}{dz} = \left(\frac{m g}{A E}\right) z = \beta z$$

Integrate

$$\int_0^{l_T} dl = \beta \int_0^L z\, dz = \frac{\beta L^2}{2}$$

Total extension $l_T = \frac{\beta L^2}{2}$ (a)

From the table, the total stretch or extension is given when suspended in water, i.e. there is a buoyancy effect.

$\therefore \quad m = (m^+ - \rho_L A)$ or mass/length minus buoyancy per length

$A = \pi(r_o^2 - r_i^2)$ — x-sectional area

$= \frac{\pi}{4}(0.1143^2 - 0.09718^2)\ m^2$ neglecting area changes with stretching

$\rho_L = 1000\ kg/m^3$

$m^+ = 24.7\ kg/m$

$l_T = 0.1503\ m$ and $L = 1000\ m$

Substitute into (a)

$$0.1503 = \beta\left(\frac{1000^2}{2}\right) \quad ; \quad \beta = 0.3006\,(10^{-6})\ m^{-1}$$

$$m = 24.7 - 1000\,(2.8436)(10^{-3})$$

$$= 21.856 \quad kg/m$$

$$\beta = \frac{(21.856)(9.81)}{(2.8436)(10^{-3})E} = 0.3006\,(10^{-6})$$

(i) $\therefore$ The Young's Modulus, $E = 2.5084\,(10^{11})$

$$\frac{kg}{m} \cdot \frac{m}{s^2}\,\frac{m}{m^2} \text{ or Pa}$$

$\longrightarrow$

(ii) Use the table provided to predict the stretch or elongation under different situations.

Now $L = 2000\,m$ and β is unchanged.

Use eq. (a)

$$\ell_T = 0.3006\,(10^{-6})\left(\frac{2000^2}{2}\right)$$

$$= 0.6012 \quad m$$

$\longrightarrow$

This is 4 times the stretch for $L = 1000\,m$, not twice.

(iii) Referring to part (ii), the maximum local strain should be at $z = L$

$$\left.\frac{dl}{dz}\right|_{max} = \left.\beta z\right|_{z=L} = \beta L = 0.3006(10^{-6})(2000)$$

$$= 6.012\,(10^{-4})$$

This exceeds the maximum strain allowed.

When the maximum strain $= 5.6(10^{-4})$, this can be achieved locally at

$$5.6(10^{-4}) = 0.3006(10^{-6})\,z$$

or $z = 1862.9$ m above the bottom of the well.

The pipe can fail anywhere in the range

$$1862.9 < z < 2000 \text{ m}$$