NHS
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ENCH 501: Transport Processes

Mid-Term Examination, Fall 2002

Instructions: Time: 2:00 to 3:30 pm Oct 28, '02
Attempt All Questions. Open Notes & Book.
Use of calculators permitted

Problem #1 (15 points)

A crude oil high in paraffins (C_{18} to C_{45}) forms wax particles suspended in the liquid when cooled to 30°C. The wax content may be as high as 10% by volume and the particles may deposit on the walls of the pipe and plug the flow channel. Your interest is to determine the parameters of operation when unloading a tanker at a terminal and flowing such a crude into a nearby refinery.

In a particular operation, the crude is discharged into a 15 cm i.d. pipe buried 32 cm in the ground (i.e. distance from the surface to the top of the pipe). The crude is pre-heated to 80°C as it enters the pipe and the average velocity in the pipe is given as 61.25 cm/s. The surface of the ground is covered by a thin layer of snow and the temperature is recorded as -5°C.

Given the data below, how long should the pipe be such that no more than 5% of the volume of the crude oil is precipitated as wax particles at the discharge, i.e. the outlet?

Data:

- Assume the densities of the crude oil and the wax particles are the same and equal 750 kg/m^3. The heat capacity of the oil is 0.375 kJ/kg K. The heat of fusion of the paraffins is given as 210 kJ/kg.
- The thermal conductivity of the soil is 1.4 W/m K.
- During phase change, a liquid-solid mixture remains isothermal.

Problem #2 (10 points)

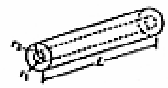
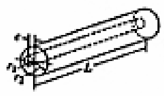
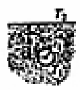
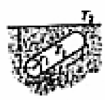
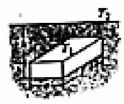
A fluid flows over a flat plate at a free-stream velocity U_∞. Derive the velocity profiles within the boundary layer using the ***integral method*** and assuming a profile of the form:

$$u = a + b \sin(cy)$$

where y is the distance from the surface of the plate into the fluid, and you have only 3 boundary conditions - $y=0$, $u=0$; $y=\delta$, $u=U_\infty$ and $\partial u/\partial y=0$.

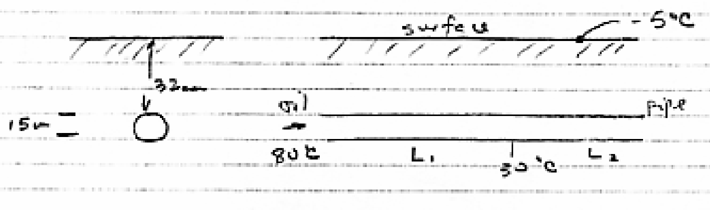
Obtain expressions for the boundary layer thickness (δ) as a function of length along the plate and for the local wall shear stress, τ_W. Compare with previously derived results.

CONDUCTION SHAPE FACTORS

Configuration	Shape Factor
Plane wall 	$\frac{A}{L}$
Concentric cylinders  $L \gg r_2$	$\frac{2\pi L}{\ln(r_2/r_1)}$ Note there is no steady-state solution for $r_2 \to \infty$, i.e., for a cylinder in an infinite medium.
Concentric spheres 	(a) $\frac{4\pi}{1/r_1 - 1/r_2}$ (b) $4\pi r_1$ for $r_2 \to \infty$
Eccentric cylinders  $L \gg r_2$	$\frac{2\pi L}{\cosh^{-1}\left(\frac{r_2^2 + r_1^2 - e^2}{2r_1 r_2}\right)}$
Buried sphere  Medium at infinity also at T_2	$\frac{4\pi r_1}{1 - r_1/2h}$ For $h \to \infty$, the result for item 3(b) is recovered
Buried cylinder  Medium at infinity also at T_2 $L \gg r_1$	$\frac{2\pi L}{\cosh^{-1}(h/r_1)}$ $\frac{2\pi L}{\ln(2h/r_1)}$ for $h > 3r_1$ For $h/r_1 \to \infty, S \to 0$ since steady flow is impossible
Buried rectangular beam  Medium at infinity also at T_2 $L \gg h, a, b$	$2.756L\left[\ln\left(1 + \frac{h}{a}\right)\right]^{-0.59}\left(\frac{h}{b}\right)^{-0.078}$

ENCH 501 Solution - Mid-Term Oct. 28, 2002

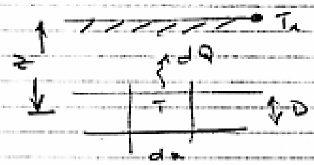
Problem # 1



As the oil flows in the pipe, it cools down to 30°C. At this temperature the wax precipitation starts.

∴ Divide the pipe into 2 section — L_1 and L_2

In the first section, consider a differential element, dx



The shape factor for the geometry is

$$S = \frac{2\pi L}{\cosh^{-1}\left(\frac{2z}{D}\right)}$$

for a differential element dx

$$dS = \frac{2\pi\, dx}{\cosh^{-1}\left(\frac{2z}{D}\right)} = \beta\, dx \qquad (2)$$

Using the shape factor method

$$dQ = k\, dS\,(T - T_\infty) = k\beta\, dx\,(T - T_\infty)$$

But this heat is extracted from the oil, i.e.

$$dQ = -\dot{m} C_p\, dT$$ where dT is the temperature drop across the element in the pipe

Hence $$k\beta\, dx\,(T - T_\infty) = -\dot{m}\, C_p\, dT$$

or $$-\frac{k\beta}{\dot{m} C_p}\int_0^{L_1} dx = \int_{T=80°C}^{30°C} d\ln(T - T_\infty) \qquad (3)$$

where $T_\infty = -5°C$, $k = 1.4 \ W/mK$

$$\beta = \frac{2\pi}{\cosh^{-1}\left(\frac{2z}{D}\right)} \quad \text{where } z = 32 + 7.5 = 39.5 \text{ cm}, \ D = 15 \text{ cm}$$

$$\therefore \ \beta = \frac{2\pi}{\cancel{1.6232}\ 2.3454} = \cancel{3.8708}\ 2.67893$$

$$m' = \dot{V}\rho = u \cdot A \cdot \rho \ , \quad \text{where } A = \frac{\pi D^2}{4} = 0.01767 m^2$$

$u = 0.6125$ m/s and $\rho = 750$ kg/m³

$$\therefore \ m' = 8.1172 \ \text{kg/s}$$

and $C_p = 375$ J/kg K (5)

Integrate equation and substitute values

$$\frac{-k\beta L_1}{m' C_p} = \ln\left[\frac{30+5}{80+5}\right]$$

$$\frac{1.4\,(\cancel{3.8708}\ 2.67893)\,L_1}{8.1172\,(375)} = \ln\left(\frac{85}{35}\right) \quad \text{or} \quad L_1 = \cancel{498.4}\ 720.145 \text{ m}$$

For the second part, the liquid with suspended wax particles is maintained at 30°C while 5% of the volume (and ∴ mass) is precipitated. That is

$$k\,S\,\Delta T = m'(0.05)\,\Delta H_{fusion}$$ (3)

where $S = \beta L_2$

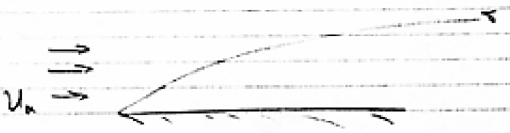
$$\therefore \ 1.4\,(\cancel{3.8708}\ 2.67893)\,L_2\,(35) = 8.1172\,(0.05)(2.1)(10^5)$$

$$\therefore \ L_2 = \cancel{449.36}\ 649.29 \text{ m}$$

Hence total pipe length $= \cancel{498.4}\ 720.145 + \cancel{449.36}\ 649.29$

$$= \cancel{947.77}\ 1369.434 \text{ m}$$ (2)

Problem #2



from Notes, p. 88, eq. 5.13, the momentum integral equation is

$$\mu \frac{\partial u}{\partial y}\bigg|_{y=0} = \frac{d}{dx}\left[\int_0^{\delta} \rho (U_\infty - u)\, u\, dy\right] \qquad (2\text{-}1)$$

Given the conditions: $y = 0$, $u = 0$

$y = \delta$, $u = U_\infty$

$y = \delta$, $\frac{\partial u}{\partial y} = 0$

Assume a profile

$$u = a + b \sin(cy) \qquad (2\text{-}2)$$

with $u = 0$ when $y = 0$ $\Rightarrow$ $a = 0$

$$\frac{\partial u}{\partial y} = bc \cos(cy)$$

using the remaining 2 b.c

$$U_\infty = b \sin(c\delta) \qquad (i)$$

$$0 = bc \cos(c\delta) \qquad (ii)$$

Alternate: from (ii), neither b nor c can $= 0$ $\therefore \cos(c\delta) = 0$ so $c\delta = \pi/2$ $\therefore c = \frac{\pi}{2}\frac{1}{\delta}$ subst. into (i) $b = U_\infty$

Multiply (i) by c and square both sides.

Square both sides of (ii). Add the 2 equations

$$(U_\infty c)^2 = (bc)^2 \{\sin^2(c\delta) + \cos^2(c\delta)\}$$

The term in bracket {} equal 1

$\therefore$ $b = U_\infty$

and $\sin(c\delta) = 1$ or $c\delta = \pi/2$

Substitute into eq. 2-2

$$u = U_\infty \sin\left(\frac{\pi}{2} \, y/\delta\right) \quad \text{where } \delta = \delta(x) \qquad (2\text{-}3)$$

Apply eq. (2-3) in eq. (2-1)

$$\frac{\partial u}{\partial y} = U_\infty \frac{\pi}{2\delta} \cos\left(\frac{\pi}{2} \frac{y}{\delta}\right)$$

$$\mu \left.\frac{\partial u}{\partial y}\right|_{y=0} = \mu \frac{U_\infty \pi}{2\delta}$$

R.H.S. of 2-1

$$\rho U_\infty^2 \int_0^\delta \left(1 - \frac{u}{U_\infty}\right)\left(\frac{u}{U_\infty}\right) dy = \rho U_\infty^2 \delta \int_0^1 \left(1 - \frac{u}{U_\infty}\right)\left(\frac{u}{U_\infty}\right) \frac{dy}{\delta}$$

Let $\eta = y/\delta$

$$\int_0^1 \left(1 - \sin\left(\frac{\pi}{2}\eta\right)\right) \sin\left(\frac{\pi}{2}\eta\right) d\eta =$$

$$\int_0^1 \sin\left(\frac{\pi}{2}\eta\right) d\eta - \int_0^1 \sin^2\left(\frac{\pi}{2}\eta\right) d\eta =$$

$$\left. - \frac{\cos\left(\frac{\pi}{2}\eta\right)}{\pi/2} \right|_0^1 - \left\{ \frac{\eta}{2} - \frac{\sin(\pi\eta)}{2\pi} \right\} \Big|_0^1$$

$$0 + \frac{2}{\pi} - \frac{1}{2} = 0.1366$$

$$\therefore \quad \mu \frac{U_\infty \pi}{2\delta} = 0.1366 \, \rho U_\infty^2 \frac{d\delta}{dx}$$

$$\frac{\nu}{U_\infty}(11.5) = \delta \frac{d\delta}{dx} \qquad \text{subject to } x = 0, \ \delta = 0$$

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$$\frac{1}{2}\int_0^\delta d\delta^2 = \frac{\nu}{U_\alpha}(11.5)\int_0^x dx$$

$$\delta^2 = 22.998\,\frac{\nu x}{U_\alpha} \quad \text{or} \quad \delta = 4.796\sqrt{\frac{\nu x}{U_\alpha}}$$

Compare : Exact coefficient = 5 ; Polynomial approx. = 4.64 | Thus is better than polynomial

Wall shear, τ_w is given by

$$\mu \left.\frac{\partial u}{\partial y}\right|_{y=0} = \mu U_\alpha \frac{\pi}{2}\frac{1}{\delta}$$

Substitute for δ

$$\tau_w = \frac{\pi}{2}\mu\frac{1}{4.796}\sqrt{\frac{U_\alpha}{\nu x}}\cdot U_\alpha$$

$$= 0.3275\,\frac{\mu U_\alpha^{3/2}}{(\nu x)^{1/2}}$$

Compare to polynomial, coeff. = $\frac{3}{2}\frac{1}{4.64} = 0.3233$

and Exact is 0.332

Again the sin profile is better than polynomial.