The University of Calgary
Department of Chemical & Petroleum Engineering

ENCH 501: Mathematical Methods in Chemical Engineering

Final Examination

Time: 8:00-11:00 a.m. **December 12, 2000**

Instructions:

Attempt all questions
Open Book, Open Notes Examination
Use of Calculators permitted

Problem #1 (25 points)

Cooling towers are used to remove waste heat from process water streams. In the operation, direct and intimate contact is maintained between water and ambient air through either atomizing the water or flowing it over flat surfaces to obtain thin films. Heat is transferred from the liquid into the air by evaporation of a small amount of the water (extraction of latent heat) and by sensible (convective and conduction) heat transfer. About 80% of the heat exchange is by the evaporation when the water is clean. However, water is often contaminated with oil in the petroleum industry. Under this condition, a thin film of oil may cover dispersed water. This inhibits evaporation and lowers the efficiency and effectiveness of a cooling tower. This is to be investigated. A limiting case when vaporization is totally suppressed is to be evaluated.

Six (6) million US gallons per hour of water at 56°C is to be cooled to 18°C before being reused. The cooling tower consists of 5 parallel vertical wooden slats over which the water is distributed to form films. The liquid flows downwards on both sides of a slat and is in contact with air at 5°C flowing upwards between the slats. The horizontal length of each slat is 1m. The heat transfer coefficient is 25 W/m^{2}K between the air and the water. Wood is a poor thermal conductor.

(a) How thick is the liquid layer on each side of a slat?

(b) Derive the equations that when solved would allow you to estimate the height of the slats required to achieve the objective above.

Data: Assume properties of water are constant at: ρ = 996 kg/m^3; μ = 0.86 mPa.s; k = 0.614 W/m K; and C_p = 4.179 kJ/kg K.

Problem #2 (25 points)

Microorganisms are to be used to digest and remove low concentrations of a toxic organic compound produced at a plant. The chief engineer has suggested collecting the plant's waste water which contains the compound in a pond at the site. The pond has a bottom lining such that nothing seeps into the ground below. The microbes are mixed with the water and they remain uniformly dispersed in the waste water. To digest the compound, the microbes also need oxygen, i.e. the microorganisms are aerobic. Oxygen dissolves from air at the surface of the pond, and as it diffuses in the liquid it is consumed at a rate which is first order with respect to the concentration of oxygen, i.e. $r_B = -k_1C_B$; $B=O_2$. It is desired to estimate the maximum depth of the pond such that some oxygen can at least reach the bottom, say at 1% of the surface concentration. The atmospheric pressure is 680 mm Hg.

(a) If k_1=0.08 s^{-1}, what maximum pond depth would you specify?

(b) What is the steady state rate of O_2 uptake per unit surface area of the pond?

Data: The diffusivity of oxygen in water is 5 (10^{-5}) m^2/s. Molar concentration of water is 55.6 moles/litre. Henry's constant for O_2 in water = $4.38(10^4)$ atm. Oxygen is 21% v/v in air.

Problem #3 (25 points; each of **a-e** has equal value**)**

(a) Two equal volumes of water (say ½ litre) are to be heated in a microwave oven. One container is styrofoam and the other is a ceramic mug. If alternately the containers are placed on one spot in the microwave, the power level and duration of activation kept identical (say 2 minutes), will the temperature of the water be the same for both systems at the end of heating? Explain.

(b) Demonstrates the Russian Peasant Method of Multiplication for the product 86 x 57.

(c) Hollow pipes with a drill bite attached to the lower end are used to drill for oil or water. Given 15 cm o.d. steel pipes with walls 1 cm thick, estimate the maximum depth of the hole that can be drilled without losing the string (series of pipes screwed together) due to metal failure past the elastic limit.

(density of steel = 7800 kg/m^3 ; Young's modulus, E = 206.8 GPa;
Maximum strain, ϵ_{max} = 0.001).

(Note: A string is made by attaching lengths of pipes, drilling until the last pipe almost disappears, pulling the string up and attaching another pipe).

(d) Explain hydroplaning from your knowledge of Couette flow.

(e) How would you go about identifying the dimensionless groups required to study the atomization of a liquid at a nozzle?

Problem #4 (25 points)

A large flat plate is totally submerged in a very large reservoir of water. Initially the plate and the water in the reservoir were stationary. Then suddenly the plate is pulled through the water at a steady speed of 5 m/s. If the density of water is 998 kg/m^3 and the viscosity is 1.6 mPa.s, use ***the integral method*** to determine:

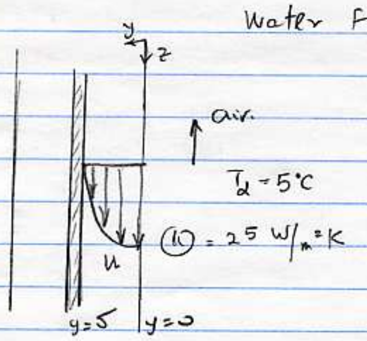
(a) The velocity distribution as a function of distance from the plate and the time elapsed since the drag was initiated.

(b) The time at which the drag (force) per unit area of the plate becomes ½ of its value 1 minute after pulling started.

Hint: If ***y*** is distance away from the plate and ***x*** is direction along which the plate is pulled, velocity in the ***x***-direction is a function only of ***y*** and time, ***t***. That is, ***u***(y,t). A differential element of volume dxd***y*** at a distance ***y*** from the plate has a momentum of ρudxd***y***. Perform a force balance for a zone from ***y***=0 to ***y***=H (H>δ) above the plate, using Newton's law.

ENCH 501 Solution FINAL EXAM Dec. 12, 2000

Problem # 1



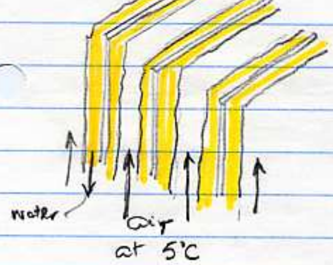
Water Flow rate = 6 million US gallons / hr

$= 6(3.785)(10^{3})\ m^3/hr$

$= 6.3083\ m^3/s$

The flow rate per metre of slat, on one side is :

$Q = 0.63083\ m^3/s$



From the Notes, p. 134, eq. 6.7

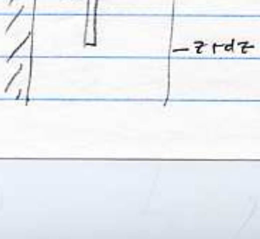
$$Q = \bar{u}\delta = \frac{\rho g \delta^3}{3\mu}$$

Using the data provided, the thickness of the film, δ, is given by

$$\delta = \left[\frac{(0.63083)(3)(0.86)(10^{-3})}{996(9.81)}\right]^{\frac{1}{3}}$$

$= 0.005531\ m$ or $5.531\ mm$

For heat transfer, consider a differential element dy by dz in the film. Perform an energy balance.



The process is steady state. Use unit width of plate.

Input $(u\,dy)\rho C_p T\Big|_z$ +

$\left(-k\frac{dT}{dy}\right) dz\Big|_y$

Output $\quad u\,dy\,\rho C_p T \Big|_{z+dz} + \left(-k\frac{dT}{dy}\right) dz \Big|_{y+dy}$

Hence

$$u\rho C_p \frac{\partial T}{\partial z} - k\frac{\partial^2 T}{\partial y^2} = 0$$

or

$$u\frac{\partial T}{\partial z} = \alpha \frac{\partial^2 T}{\partial y^2} \quad ; \quad \alpha = \frac{k}{\rho C_p} \qquad (1\text{-}1)$$

Subject to $\quad z=0 \quad T = T_e = 56\,°C$

$y = 0 \quad -k\frac{dT}{dy} = h(T_\alpha - T) \quad ; \quad T_\alpha = 5\,°C$

$y = \delta \quad \frac{\partial T}{\partial y} = 0 \quad$ (insulator)

and equation 6.4 with $\beta = 0°$

$$u = \frac{\rho g \delta^2}{2\mu}\left(1 - \left(\frac{y}{\delta}\right)^2\right) \qquad (1\text{-}2)$$

Equation 1-1 (with eq. 1-2) is to be solved to obtain $T(y, z)$.

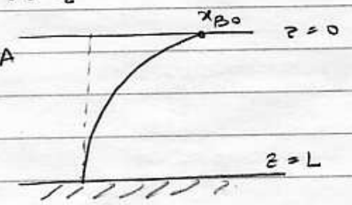
The bulk mean temperature at any distance z is given by

$$T_{bm}(z) = \frac{\int_0^\delta u T(y,z)\,dy}{\int_0^\delta u\,dy} \qquad (1\text{-}3)$$

The question is: ~~At~~ what z is $T_{bm} = 18\,°C$?

Problem #2

$B \equiv O_2$



This problem is as on pages 175 and 176 of Notes. It involves diffusion with chemical reaction w.r.t. O_2.

First the O_2 dissolves at the interface.

Use Henry's Law, $y_B P = \bar{P}_B = H x_{B0}$

$$x_{B0} = \frac{(0.21)(680/760)}{4.38(10^4)} = 4.2898(10^{-6})$$

At steady state, the material balance equation on oxygen is

$$\frac{dN_B}{dz} + k_1 C_B = 0$$

The flux is given as:

$$N_B = -D_{AB} C \frac{dx_B}{dz} + x_B (N_A + N_B)$$

It is obvious that $x_B \le x_{B0}$ is small. Hence drop second term on the right. The material balance equation is hence

$$-D_{AB} \frac{d^2 x_B}{dz^2} + k_1 x_B = 0 \quad ; \quad C = \text{constant}$$

$$(C_B = C x_B)$$

This equation is subject to

$z = 0 \qquad x_B = x_{B0}$

$z = L \qquad \dfrac{dx_B}{dz} = 0 \qquad$ (impervious wall)

The solution – eq. 122 in Notes is

$$\frac{x_B}{x_{B0}} = \frac{\cosh \Gamma(1 - z/L)}{\cosh \Gamma} \quad ; \quad \Gamma = \left[\frac{k_1 L^2}{D_{AB}}\right]^{1/2}$$

(a)

For the problem, when $z = L$, we want O_2 to be at least present – say $0.01\, x_{B0} = x_B$, i.e. 99% approach. Hence

$$0.01 = \frac{\cosh(0)}{\cosh \Gamma} = \frac{1}{\cosh \Gamma}$$

$$\cosh \Gamma = 100 \quad \Rightarrow \quad \Gamma = 5.2983$$

$$\therefore \quad 5.2983 = \left[\frac{0.08\, L^2}{5(10^{-5})}\right]^{1/2} \quad \Rightarrow \quad L = 0.1325\ m$$

The pond should be about 13.25 cm deep. $\longrightarrow$

(b) From equation 6.124

$$N_B\Big|_{z=0} = \left(\frac{D_{AB}\, c\, x_{B0}}{L}\right) \Gamma \tanh \Gamma$$

$$= \frac{5(10^{-5})(55.6)(10^3)(4.2898)(10^{-6})}{0.1325} \times 5.2983 \times 0.99995$$

$$= 4.77(10^{-4}) \quad moles/m^2 \cdot s \longrightarrow$$

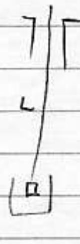
Problem #3

(a) No. The temperature of the water will be higher in the styrofoam cup than in the ceramic mug. Note that microwave radiation interacts primarily with water molecules – not the polystyrene (of foam cup) or the ceramic.

As energy is absorbed by water in the containers and transformed into heat, the ceramic – with its higher conductivity and greater thermal capacity than those for styrofoam, withdraws and stores some of the heat from the water.

(b)

~~57~~	× ~~86~~	Double left and $\frac{1}{2}$ right until r.h.s. equals 1. Cross out terms with even r.h.s. (Record only whole numbers on r.h.s.)
114	43	
228	21	
~~456~~	~~10~~	
912	5	
~~1824~~	~~2~~	
3648	1	
Sum 4902		

(c) 

This is on limits.

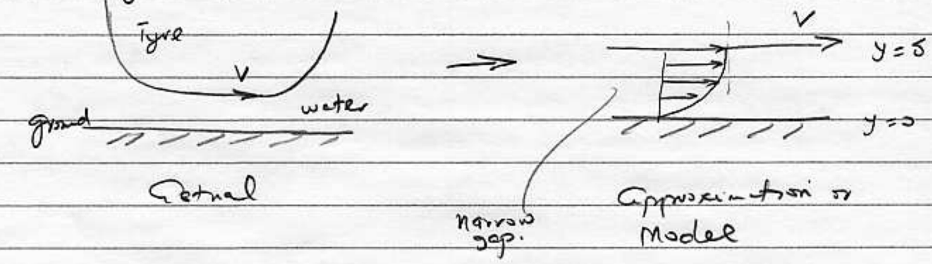
$$\frac{m^{*} g L}{A} < E\,\varepsilon_{max}$$

$$\rho g L < E\,\varepsilon_{max}$$

$$L < \frac{E\,\varepsilon_{max}}{\rho g} = 2702.6 \; \underrightarrow{m}$$

m^{*} = mass / unit length
L = string length
A = X-sectional area
E = Young's modulus
ε_{max} = max. strain.

(d) Hydroplaning — moving without friction on a layer of water.



from Notes: p 145-6, velocity distribution is:

$$\frac{u}{V} = \frac{y}{\delta} + \Gamma \frac{y}{\delta}\left(1 - \frac{y}{\delta}\right) \; ; \; \Gamma = -\frac{\delta^2}{2\mu}\left(\frac{x}{L}\right)\frac{1}{V}$$

shear stress

$$\tau = -\mu \frac{du}{dy} \; ; \; \frac{du}{dy} = V\left[\frac{1}{\delta} + \frac{\Gamma}{\delta} - \frac{2\Gamma y}{\delta^2}\right]$$

$\tau = 0$ at $y = \delta$ when $\Gamma = 1$ $\longrightarrow$

(e) A home-work problem.

Diameter of droplets = f (Diameter of orifice, velocity, density, liquid viscosity, surface tension)

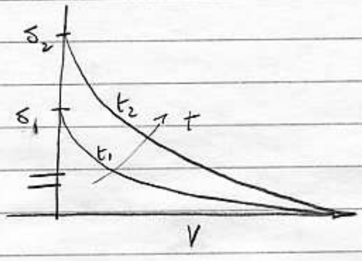
or $d = f(D, u, \rho, \mu, \sigma)$

3-dimensions — L, M, t $\therefore$ 3 dimensionless gps.

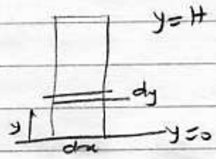
use Buckingham Π theorem

$$\frac{d}{D} = \phi\left[\frac{\sigma\rho^2 u^3}{\mu^3} \; ; \; \frac{u\rho d}{\mu}\right]$$

Problem # 4



Consider one side of the plate.
Penetration depth = δ



(a)

Consider a block above the plate.

For a differential element dy — mass = $dx\,dy\,\rho$ — unit depth

If this has a velocity u, its momentum is:

$$\rho u\, dx\, dy$$

Net momentum above plate is $\left[\int_0^H \rho u\, dy\right] dx$

The rate of change is

$$\frac{d}{dt}\left[\int_0^H \rho u\, dy\right] dx = \text{Net Force}$$ | Newton's Law

The Net force is shear stress or wall shear

$$\tau_w\, dx = -\mu \left.\frac{du}{dy}\right|_{y=0} dx$$

Hence the force balance equation is

$$\frac{d}{dt}\left[\int_0^H \rho u\, dy\right] = -\mu \left.\frac{du}{dy}\right|_{y=0} \qquad (4\text{-}1)$$

where $u(y,t)$ is unknown.

Boundary conditions for the problem are:

$y=0,\ u=V\ ;\ y=\delta,\ u=0\ ;\ y=\delta,\ \frac{du}{dy}=0$

By Integral method — assume a profile

$$u = a + by + cy^2$$

Use b.c. $a = V$, $b = -\frac{2V}{\delta}$ and $c = \frac{V}{\delta^2}$

$$\therefore \quad \frac{u}{V} = \left(1 - \frac{y}{\delta}\right)^2 \qquad (4-2)$$

Let $\eta = y/\delta$. Substitute eq. 4-2 into 4-1

$$\frac{d}{dt}\left[\int_0^H V(1-\eta)^2 \delta\, d\eta\right] = \frac{\mu}{\rho}\frac{2V}{\delta} \qquad (4-3)$$

Set upper limit of integral as δ, i.e. $H=\delta$

$$\frac{d}{dt}\left[V\delta \int_0^1 (1-\eta)^2 d\eta\right] = \frac{2\nu V}{\delta}$$

Hence $\quad \delta\, d\delta = \frac{1}{2} d\delta^2 = 6\nu\, dt$

$$\delta = \sqrt{12\nu t}$$

Hence velocity profile is $\quad \frac{u}{V} = \left[1 - \frac{y}{\sqrt{12\nu t}}\right]^2$ →

(b) $$\frac{\tau_w|_t}{\tau_w|_{1min}} = \frac{\mu \frac{du}{dy}|_0|_t}{\mu \frac{du}{dy}|_0|_{1min}} = \frac{\delta|_{1min}}{\delta_t} = \left[\frac{1}{t}\right]^{\frac{1}{2}}$$

Given $\frac{\tau_w|_t}{\tau_w|_{1m}} = \frac{1}{2}$, $\quad t = 4$ min →