Opportunistic Transmission using Large Scale Channel Effects

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Abstract

This paper quantifies the multi-user wireless diversity benefit of basing opportunistic transmission on large scale shadowing. This technique is primarily aimed at wireless sensor nodes. The processing capability of these nodes is limited and transmissions between sensor nodes are very sporadic. It is therefore easier for these nodes to track large scale shadowing variations than the small scale shadowing variations normally used for opportunistic transmission. This paper will derive closed form expressions that compare the average SNR improvement provided by opportunistic transmission when based on large scale and small scale channel effects. Closed form bit error rate (BER) expressions will also be used to illustrate that opportunistic transmission based on shadowing can achieve very good performance improvements for modestly sized scheduling groups and realistic indoor shadowing standard deviation values.

I. INTRODUCTION

It is well established that opportunistic transmission improves performance in a multi-user wireless channel by providing multi-user diversity [1]. Wireless sensor networks (WSN’s) would benefit from this form of diversity but the design constraints of WSN nodes present a challenge.

Typically, an opportunistic transmission scheme will track small scale fading in the signals received from each user and use that information to make scheduling decisions. However, many sensor applications require very small devices that are able to operate for a long time without changing or recharging batteries. This means sensor nodes will utilize frequent sleep cycles, have limited computational capability and will transmit to each other only sporadically. All of these factors make it difficult to track the rapid variations in small scale fading.

The solution proposed in this paper is to base opportunistic transmission decisions on large scale shadowing variations. This will be referred to as large scale multi-user diversity (LSMD).
While small scale fading will change drastically after moving only a fraction of the carrier wavelength, shadowing changes occur over several meters [2], [3]. For carrier frequencies in the GHz range, this means that a WSN using opportunistic transmission would be able to reduce channel estimation sampling rate by approximately two orders of magnitude.

One potential disadvantage of LSMD is latency. The slow rate of change of the shadowing process would mean that individual users would experience long delays between being allowed to transmit. However, latency is not a concern for many WSN applications. For example, updates for vital signs such as blood pressure or body temperature are required only every few seconds. Measurements show that shadowing fluctuates on approximately this same timescale for indoor pedestrian users [2], [3]. In addition, WSN applications with lower latency requirements could use a proportionally fair scheduling scheme that accounts for throughput [1].

Basing radio resource allocation decisions on shadowing has been explored by other authors. In [4], ergodic capacity is used to show that ignoring small scale fading and basing opportunistic user selection on large scale shadowing conditions does provide a performance benefit. However, their results do not provide any insight into the system bit error rate (BER) performance achieved by this scheme at realistic signal to noise ratio (SNR) values. The authors in [5] also investigate multi-user diversity based on random pathloss values from an information theory perspective. In [6], large scale effects are used for power allocation rather than scheduling decisions. The authors in [7] also investigate resource allocation when only knowledge of large scale shadowing is known but no expression is provided that characterizes overall system performance.

The first contribution of this paper is to derive a closed form expression for the average SNR diversity gain provided by LSMD. This expression will show that LSMD compares very favorably with opportunistic transmission based on small scale fading. The second contribution is the derivation of a closed form expression for BER in an LSMD system that further quantifies the benefit of LSMD for different user population sizes and channel conditions. These expressions are based on using a single average SNR value to summarize conditions for a group of users with different positions. Simulations will be used to illustrate the scenarios where this approximation yields an accurate prediction of BER.

Section II defines the wireless system being analyzed and outlines the protocol that would be used to implement LSMD. Section III derives the multi-user diversity SNR improvement achieved by LSMD and compares it to conventional small scale opportunistic transmission. Section IV
presents the derivation of the BER expression that characterizes how system performance is improved by LSMD. Finally, concluding remarks are made in Section V.

II. SYSTEM MODEL

A system is assumed where $K$ wireless nodes transmit directly to a central node. This can represent nodes communicating with a central base station or access point. It also applies to an ad-hoc WSN where the nodes are organized into clusters [8]. The simplest form of opportunistic transmission is assumed where the central access point or cluster head node simply selects the node with the best channel quality for transmission. This is in line with the goal of simplifying the processing load on the WSN cluster head as much as possible. As with most opportunistic transmission schemes, it is assumed that the nodes in the network are mobile.

A time division multiplexing and duplexing scheme is assumed where the same carrier frequency is used for the forward and reverse link. In its simplest form, the protocol used to implement LSMD would add an additional field to the acknowledgment (ACK) messages sent out by the central node that would indicate who should transmit next. All nodes would then monitor each ACK message and transmit when requested. LSMD could also be implemented using the guaranteed time slot (GTS) mechanism described in the 802.15.4 standard [9].

Given the narrowband nature of most WSN applications, a flat fading channel is assumed. At the central access point or cluster head, the $i$th symbol received from node $k$ is

$$r_k(i) = c_k(i)\sqrt{Y_k(i)A_k}x_k(i) + N_k(i)$$

(1)

where $c_k(i)$ is the zero mean complex Gaussian process representing Rayleigh small scale fading, $Y_k(i)$ is a log-normal distributed large scale shadowing process, $A_k$ is an attenuation factor that represents distance dependent path-loss, $x_k(i)$ is the $i$th uncoded BPSK symbol transmitted to user $k$ and $N_k(i)$ is zero mean complex Gaussian noise with power spectral density $N_0$. When the shadowing process is expressed in dB, it has a mean and standard deviation of $\mu_L$ and $\sigma_L$, respectively. It will be assumed that $\mu_L = 0$ dB and $\sigma_L$ will be varied to reflect different environments.

It is assumed that both the fading processes, $c_k(i)$, and the shadowing processes, $Y_k(i)$, are independent and identically distributed for each user. Since the decorrelation distance for indoor
shadowing is approximately 2 m, this corresponds to a scenario where nodes are separated by at least a few meters. In a medical setting, this could correspond to sensors on patients in a busy waiting room or hallway.

III. Multi-user Diversity SNR Improvement

In this section, analysis is used to compare how much opportunistic transmission improves the average received SNR of a system when its scheduling decisions are based on either small scale or large scale channel effects. The instantaneous SNR of the signal received from the \( k \)th node can be defined as

\[
\gamma_k = \frac{E_b}{N_0} A_k C_k Y_k
\]

(2)

where \( C_k \) and \( Y_k \) represent the small and large scale power gain factors of the channel, respectively, and \( A_k \) is defined in Section II as path loss attenuation. Assuming the Rayleigh fading conditions typical of an indoor environment, \( C_k \) is exponentially distributed and it is assumed that \( E \{ C_k \} = 1 \). When \( Y_k \) is expressed in dB, \( E \{ Y_k \} = \exp[\mu_L/(10 \log_{10} e) + (\sigma_L/(10 \log_{10} e))^2]/2] \).

The value of \( A_k \) will vary randomly as user \( k \) moves about the environment. However, it is assumed that all sensor nodes remain in the same local area such that \( E \{ A_k \} \) is equal to the same value, \( E \{ A \} \), for all \( k \). In a hospital setting, this would typically occur when several patients are moving about the same waiting room or hospital ward.

When using small scale channel effects to make scheduling decisions, it is assumed that the instantaneous SNR values used for these decisions have been processed with a normalizing filter to remove large scale channel effects [10]. This sort of filter uses a time constant much longer than the coherence time in order to produce the estimate of \( A_k Y_k \) that is required for this normalization.

As a result, this small scale scheduling scheme is equivalent to a selection diversity system with \( K \) branches that experience independent Rayleigh fading. Using the average SNR expression from [11] for this form of diversity, the average SNR for scheduling based on normalized small scale fading statistics can be written as

\[
E \{ \gamma(K) \}_{SS} = \frac{E_b}{N_0} E \{ A \} \sum_{k=1}^{K} \frac{1}{k}
\]

(3)
with the multi-user average SNR diversity gain expressed as

$$\rho_{SS}(K) = \frac{E\{\gamma(K)\}_{SS}}{E\{\gamma(1)\}_{SS}} = \sum_{k=1}^{K} \frac{1}{k}. \quad (4)$$

The SNR multi-user diversity gain of LSMD can be found by assuming that LSMD scheduling calculates SNR using a time constant long enough to average out small scale fading. The path loss gain values $A_k$, which are assumed to vary on a timescale even longer than shadowing, are normalized out in a method similar to the way $A_k Y_k$ is removed by the small scale scheduler. This allows the LSMD scheduler to base its decisions solely on $Y_k$.

LSMD selects the node with the maximum $Y_k$ for transmission. Since the $Y_k$ gains are assumed to be independent and identically distributed, the PDF of $Y_{\text{Max}} = \max\{Y_1, Y_2, \ldots Y_K\}$ is given by [12]

$$f_{\text{Max}}(y, K) = K f(y) F^{K-1}(y) \quad (5)$$

where $f(y)$ and $F(y)$ are the probability density function (PDF) and cumulative distribution function (CDF) of the lognormally distributed channel gains.

For a given large scale channel gain $Y$, (2) can be used to show that the average SNR of a signal received from a node is $E\{\gamma\}_{LS|Y_k=Y} = (E_b/N_0) E\{A\} Y$ since $E\{C_k\} = 1$. The overall average SNR achieved by LSMD can then be determined by averaging $E\{\gamma\}_{LS|Y_k=Y}$ over $f_{\text{Max}}(y)$ such that

$$E\{\gamma(K)\}_{LS} = \frac{E_b}{N_0} E\{A\} \int_0^\infty y f_{\text{Max}}(y, K) dy. \quad (6)$$

In order to find a closed form solution for (6), the logistic distribution is used as an approximation to the log-normal PDF and CDF [13] such that the PDF and CDF of $Y_k$ would be written as

$$f(y) = py_m y^{p-1} \left[1 + \left(\frac{y_m}{y}\right)^p\right]^{-2} \quad (7)$$

$$F(y) = \left[1 + \left(\frac{y_m}{y}\right)^p\right]^{-1} \quad (8)$$
where \( y_m = \exp \left[ \frac{\mu_L}{(10 \log_{10} e)} \right] \) and \( p = \pi 10 \log_{10} e/ (\sigma_L \sqrt{3}) \).

Substituting (7) and (8) into (5) gives the following expression

\[
f_{\text{Max}}(y) = Kpy_m^p \left( \frac{1}{y} \right)^{p+1} \left[ 1 + \left( \frac{y_m}{y} \right)^p \right]^{-2-(K-1)}.
\]  \hspace{1cm} (9)

Incorporating (9) into (6) yields a formula that can be solved in closed form according to [14, eq. 3.241-4] such that

\[
E \{ \gamma(K) \}_{LS} = \frac{E_{N_0} E \{ A \} \int_0^\infty y f_{\text{Max}}(y) dy}{E_{N_0} E \{ A \} y_m^p K \int_0^\infty y^{-p} (1 + y_m y^{-p})^{-(K+1)} dy} \hspace{1cm} (10)
\]

so that the average SNR gain provided by LSMD is written as

\[
\rho_{SS}(K) = \frac{E \{ \gamma(K) \}_{LS}}{E \{ \gamma(1) \}_{LS}} = \frac{K \Gamma [1 + K - (p-1)/p] \Gamma(2)}{\Gamma [2 - (p-1)/p] \Gamma(1+K)} \hspace{1cm} (11)
\]

A comparison of the average SNR diversity gain provided by small scale fading opportunistic transmission and LSMD is shown in Fig. 1. This plot shows both \( \rho_{SS}(K) \) and \( \rho_{LS}(K) \) expressed in dB and plotted versus \( K \) for \( \sigma_L = 2.3 \text{ dB} \). This \( \sigma_L \) reflects shadowing in an indoor environment [2], [3]. Fig. 1 also shows numerical estimates of \( \rho_{SS}(K) \) and \( \rho_{LS}(K) \) generated by simulating received SNR values for the \( K \) users according to (2) and selecting users for transmission based on large scale or small scale channel values as described above.

The results in Fig. 1 are significant since they indicate that LSMD performance is very comparable to traditional small scale opportunistic transmission. The small scale scheduling outperforms LSMD since the statistical variations of Rayleigh fading exceed that of the shadowing process. However, the performance of LSMD is still very good, particularly considering the very modest value assumed for \( \sigma_L \). This means LSMD is an attractive option for implementing multi-user diversity with a lower channel estimation sampling rate. Fig. 1 also indicates that the approximation in (11) is reasonably accurate.

The slight disagreement between the numerical and analytical predictions of average LSMD SNR gain is due to the logistic approximation used for the log-normal distribution. The disagreement becomes apparent because the numerical results are generated with a true log-normal
The derivation of a BER expression for LSMD begins by noting that the average probability of bit error for a BPSK system subject to Rayleigh fading can be approximated as $P_b(\gamma) = 1/(4E\{\gamma\})$ where $E\{\gamma\}$ is average SNR [15]. As discussed in Section III, average SNR in an LSMD system for a given shadowing gain, $Y$, can be written as $E\{\gamma\}_{LS}|_{Y_k=Y} = E_b/N_0E\{A_k\}Y$ such that the probability of bit error experienced by user $k$ for a given $Y$ is

$$P_b(Y) \approx \frac{1}{4E_b/N_0E\{A_k\}Y}. \tag{12}$$

As in Section III, it is assumed that $E\{A_k\} = E\{A\}$ for all $k$. The effect of this assumption will be evaluated in this section by comparing this analysis to numerical BER results that incorporate users at different positions with correspondingly different values of $A_k$.

Since $Y$ in an LSMD system will be distributed according to (5), the average probability of bit error as a function of user population $K$ can be determined by integrating (12) over (5) such that

$$P_b(K) = \int_0^\infty P_b(y)f_{Max}(y, K)dy \tag{13}$$

Again, the logistic approximation of the log-normal PDF and CDF is used to determine $f_{Max}(y, K)$. Substituting (9) into (13) results in an integral that can also be solved according to [14, eq. 3.241-4] such that the following closed form solution can be obtained

$$P_b = \frac{Ky_m^p}{4E_b/N_0E\{A\}} \left( \frac{1}{y_m^p} \right)^{\frac{p+1}{p}} \frac{\Gamma[(p + 1)/p] \Gamma[2 + K - 1 - (p + 1)/p]}{\Gamma(2 + K - 1)} \tag{14}$$

Note that this expression uses a single average SNR value, $E\{A\} E_b/N_0$, to approximate conditions for a network with multiple users at different positions. It is assumed that all users are located within a circle of radius $r$ and that the distance between the base station or cluster head node and the center of that circle is $d_o$. The value of $E\{A\}$ in (14) can then be interpreted as the path loss at distance $d_o$. 

IV. Probability of Bit Error
It should also be noted that the diversity order of this system is one since $E_b/N_0$ in (14) is not raised to an exponent. As a result, the terms in (14) that are a function of $K$ will linearly scale $E_b/N_0$. As $K$ increases, this means that LSMD results in a power gain, not an increase in diversity order.

To verify (14), simulations are conducted that calculate the average BER experienced for $K$ LSMD users located within a circle of radius $r$. Independent and log-normally distributed shadowing profiles are generated for each of the $K$ mobiles. The shadowing profiles are then multiplied by zero mean, unit power complex Gaussian sequences to model Rayleigh fading. For each symbol, the user with the maximum shadowing value is selected for uncoded BPSK transmission.

During the simulation, the positions of the users are randomly and uniformly redistributed inside the circle of radius $r$ for each new data symbol. The distance from the center of the circle to the cluster head node, $d_o$, is normalized to 1. If $d_k$ is the distance from user $k$ to the cluster head, then $A_k = (d_o/d_k)^3$, where a pathloss exponent of 3 is chosen for the indoor environment. Note that this means $E\{A\} = 1$ in (14).

On the simulation plots, BER on the y-axis represents the average BER experienced by all users. SNR on the x-axis represents the average SNR in the center of the circle containing the users. This SNR value is scaled by $A_k = (d_o/d_k)^3$ to calculate the SNR conditions experienced by individual users.

Fig. 2 shows analytical and numerical BER for varying group size. For this figure, it is assumed that $r = 0.2$. This gives a $d_k/d_o$ distance ratio comparable to users deployed in a 10 m wide room that communicate with a cluster head that is 25 m away. For this curve, $\sigma_L = 2.3$ dB, as in Section III. As expected, the diversity gains shown in Fig. 2 agree with the values predicted in Fig. 1. Both of these plots show that the number of users can be kept modest and still realize good diversity gain with LSMD. This is important since the load on the cluster head node coordinating the scheduling should be minimized.

Fig. 2 also confirms that the LSMD diversity gain manifests itself as a power gain rather than a change in the slope of the BER curve. While this evident from examination of (14), the power gain can also be understood by noting that small scale effects are ignored by the LSMD scheduling decisions. This means that LSMD link selections do not affect the small scale Rayleigh component of the overall diversity channel. Choosing the diversity link with the
best shadowing value does improve average received SNR but the slope of the BER curve is not
affected primarily because the Rayleigh small scale channel statistics remain unchanged.

Fig. 2 also verifies the accuracy of the analytical result in (14). This includes the assumption
that $E \{ A_k \}$ can be assumed to be approximately the same for all $K$ users under reasonable
deployment conditions. The disagreement between the analytical and simulated results in Fig. 2
at low SNR values primarily comes from the probability of error approximation in (12) which
is known to be most accurate in the high SNR region.

It is also important to evaluate the accuracy of using a single average SNR value in (14) to
represent conditions for a group of users with different pathloss values. It is expected that the
accuracy of this assumption will degrade for increasing values of $r/d_o$. Fig. 3 compares (14) to
simulation results for $\sigma_L = 2.3$ dB, $K = 4$ and $r$ values ranging from 0.2 to 0.8. As an example,
for an unnormalized value of $d_o = 25$ m, these values of $r$ would correspond to circle radii
ranging from 5 m to 20 m. The results in Fig. 3 indicate that (14) remains accurate for coverage
area radii equal to slightly greater than 40% of the distance between the circle center and the
base station.

In many scenarios, the assumption of uncorrelated fading made in the derivation of (14) will
only be an approximation. The work in [3] and the references therein establish that indoor
shadowing is uncorrelated for a user separation of approximately 2 m or greater. However, that
correlation rises to approximately 61% for a 1 m separation. It is important to establish whether
the accuracy of (14) is maintained for the moderate levels of shadowing correlation that occur
for small separation distances between users.

To evaluate the accuracy of (14) when users are closely spaced, the results in Fig. 2 are
regenerated using the simulation described above with a 61% correlation imposed on the shad-
owing processes experienced by neighbouring users. This means that the shadowing experienced
by user $k$ is 61% correlated with users $k - 1$ and $k + 1$, when the shadowing processes are
expressed in dB. The results, shown in Fig. 4, indicate that (14) remains within approximately
1 dB of the numerical results when correlated shadowing is introduced.

Fig. 5 shows BER performance for $K = 4$ users and shadowing standard deviation values of
1 dB, 2 dB and 4 dB. As expected, the plot indicates that multi-user diversity improves with
increased channel variation. The diversity gain is on the order of the increase in the shadowing
standard deviation. The plot also indicates that (14) remains accurate for the range of $\sigma_L$ values
that can be expected in an indoor environment.

V. CONCLUSION

To conclude, this paper has made some important observations regarding opportunistic transmission that is based on large scale shadowing. First, analytical results show that the average SNR gain provided by LSMD is very comparable to conventional opportunistic transmission based on small scale fading. This is important since LSMD can be implemented using a much lower channel estimation sampling rate than schemes that require small scale channel information.

In addition, an analytical expression for BER in an LSMD system is derived and verified using simulation. The BER results further illustrate that LSMD can provide a significant performance improvement for realistic shadowing standard deviation values and a modest number of users.
REFERENCES


Fig. 1. Average SNR gain.
Fig. 2. System performance for a variable group size.
Fig. 3. System performance for a variable coverage area.
Fig. 4. System performance for correlated shadowing.
Fig. 5. System performance for a variable shadowing standard deviation.