A Framework for Modelling Residential Prosumption Devices and Electricity Tariffs for Residential Demand Response

Marc Beaudin, Hamidreza Zareipour, Senior Member, IEEE, and Antony Schellenberg

Abstract—In this paper, we propose a unified framework for inclusion of different electricity tariff components and residential prosumption devices in an optimal energy management system (REMS). This framework allows REMS to use flexible modelling with respect to devices and price components applicable to an individual house, without requiring unique models for each device and tariff. The significance of this work is that the proposed framework facilitates the process for investigating how dwellings may respond to various tariffs when energy prosumption is optimally scheduled. In this work, we include six cost components and four response classes to illustrate the usage of this framework.

Index Terms—Residential Energy Management Systems, Residential electricity pricing models, residential load models, power scheduling, Linear Programming

I. INTRODUCTION

This paper’s primary objective is to provide a flexible framework for including electricity tariff structures and energy prosumption devices in an optimal energy management systems for the residential sector. To achieve this, we provide formulation for modelling existing electricity tariff structures and typical electrical devices in the residential setting.

The utility industry has offered residential consumers various electricity tariff structures in order to better reflect their contribution to the cost of electricity production, and to give incentives to consumers to improve their load profile through demand response. For example, in Ontario, Canada, residential electricity customers are billed on a time-of-use plan, where the cost of electricity cycles according to the time of day [1]. Another example is the Gulf Power Good Cents Select, where the utility charges an additional 0.29$/kWh for up to 1% of the year. Alternatives to changing the price structure also exist, such as Incentive Based Programs [2], which include direct load control, interruptible/curtailable programs, demand bidding, emergency demand response, capacity markets, and ancillary services market.

However, electricity is a commodity with a high value compared to its price, which results in a low will to change consumption. This is evidenced by the high value of residential load, up to 16 €/kWh in [3], to the price of electricity, in the range of 0.1 €/kWh. Thus, elasticity is low (e.g., 0.02 - 0.069 in Illinois and 0.027 - 0.054 in California [4]) unless the price of electricity is high, such as during a critical peak pricing period. However, willingness to respond may improve if the prosumption of some devices were automated.

To better respond to varying electricity prices and other signals, Residential Energy Management Systems (REMS) are proposed in [5] and [6] to automate energy prosumption, by shifting the burden of control away from the prosumer. Effective REMS consider tariffs, forecasts, and load information to schedule an optimal prosumption profile within a home area network. Some REMS base scheduling on heuristics [6]–[9], while others use time-series optimization [10]–[12]. For optimization-based approaches, varying the electricity pricing structure changes the nature of the objective problem and its constraints. For example, the Incremental Block Rate [11], Peak Demand Charge [13], and Time-of-use [14] structures require different formulation. The load models employed in REMS in the literature tend to be inconsistent, partially because of the variability in loads. In [13], [15], each device is modelled separately, while all loads follow a common formulation in [11], [16], [17]. Residential load is binned into baseline load, regular loads and burst loads in [18] to recognize the variable energy prosumption behaviour of residential loads while limiting the complexity of formulation.

In this paper, we review and discuss existing electricity tariff structures in the context of time-series optimal scheduling of REMS. We provide a new framework for scheduling prosumption by partitioning tariffs into pricing component types, such that a combination of various pricing components form electricity tariffs. Additionally, we present general models for residential devices by binning the devices into limited response classes, based on the diverse residential device models and descriptions reviewed in the literature. In the present paper, six pricing components and four response classes of devices are used to illustrate the presented framework. Mixed Integer Linear Programming (MILP) models are presented when Linear Programming (LP) models cannot accurately capture the behaviour of tariffs and loads.

There are three contributions in the proposed paper. First, we provide a modelling framework for price and load for optimal residential energy scheduling. Second, we concisely present formulation for residential tariff structures and electrical devices based on descriptions found in the literature. Finally, we expand the literature on the implication of pricing mechanisms on REMS and metering.

The rest of the paper is organized as follows: In Section II, we discuss the REMS’ approach to solving the problem. In Section III, we provide a general linear programming formulation for energy costs, critical peak pricing, peak demand charges, inclining block rates, and provide mixed-integer programming formulation for declining block rates. In Section IV, we provide models to represent the residential electricity demand. Finally, in Section V, we summarize and discuss the potential for the implementation of tariff structures.
II. METHODOLOGY

In this research, we formulate a general optimal energy scheduler based on a broad survey of the literature on REMS. The model’s objective is to minimize the overall energy costs due to electricity prosumption, from the prosumer’s perspective, while ensuring that household devices operate in a manner that maintains an expected quality of life for the occupants in the dwelling. Some examples of social factors that are relevant to quality of life include the thermal comfort of the occupants [15], the inconvenience of waiting for an energy service [11], and the value of each energy service provided [19]. The literature, however, is not consistent on reflecting such factors in REMS models. For example, the works in [20], [13], [21], [15], [11], [10], [22] consider the dwelling’s well-being and values in its scheduling objective, while the works in [23], [17], [24], [25] set permissible operational limits for devices. As such, we focus only on the cost of electricity provision in our model, although we do include reasonable limits in our optimization-based REMS in order to reflect the occupant’s expectation of well-being, such as household temperature limits. Such limits are mainly embedded in the load models, discussed in Section IV. The optimization problem solved by the REMS is as follows:

\[
\min F = \sum_{c \in C} f_c \tag{1}
\]

subject to:
\[
\begin{align*}
\xi & \quad \zeta \\
\end{align*}
\]

where \(F\) denotes the overall electricity cost and \(f_c\) represents cost components. A residential tariff structure is often comprised of several cost components, \(c \in C\), as illustrated in Table II. In this Table, \(FC\) represents fixed costs, \(FPE\) is the fixed price of energy, \(PDP\) is prosumption dependant prices, \(CPP\) is critical peak pricing, and \(PDC\) is peak demand charge. \(c\) is the index of cost components and \(C\) is the set of all cost components. i.e., \(C = \{FC, FPE, PDP, CPP, PDC\}\). For example, commercial customers of Hydro Quebec’s tariff include a fixed daily cost component (i.e., \(c = FC\)), a cost component dependent on the day’s energy prosumption (i.e., \(c = FPE\)), and a cost component based on the maximum power draw over a moving 15-minute average (i.e., \(c = PDP\)). \(f_c\) is the cost component \(c\) of the tariff structure. \(\xi\) and \(\zeta\) are the set of constraints representing electricity pricing tariffs models and the load models, respectively.

Cost component \(f_c\) may be fixed, or dependant on peak power prosumption or energy prosumption. Let \(P\) be the set of prosumption values of the dwelling, i.e., \(P = \{P_t | t \in T\}\), where \(T\) is the set of time steps in the scheduling horizon and \(P_t\) is the total prosumption of the dwelling at time step \(t\). For example, for hourly time-steps, and a 24-hour scheduling horizon, \(T\) would have 24 elements. The total prosumption at each time step \(t\), which is the sum of prosumption from each device \(d \in D\), at each time step, is as follows:

\[
P_t = \sum_{d \in D} P_{t,d} \tag{2}
\]

where \(P_{t,d}\) is the prosumption of each device \(d\). Equation (2) calculates the total prosumption of the dwelling at time \(t\).

If pricing of electricity is applied non-uniformly to various intervals in the day, then it is necessary to partition \(T\) into several time-intervals, \(T_i\). For example, if Time-of-use (TOU) pricing is applied, where energy has a unique price in the morning, afternoon, and night, then time-intervals must be created for each morning, afternoon, and night, as follows:

\[
\bigcup_{i \in I_c} T_i = T \tag{3}
\]

\[
T_i \cap T_{i'} = \emptyset \quad \forall (i, i' \in I_c) \land (i \neq i') \tag{4}
\]

where, \(I_c\) is the set of time-intervals for cost component \(c\). The indices \(i\) and \(i'\) are both elements of the set \(I_c\). Equation (3) states that the union of all time-intervals, denoted by \(T_i\) covers all the time steps in the scheduling problem, and (4) states that each time-interval is mutually exclusive. The total energy prosumption during time-interval \(T_i\) is defined, as follows:

\[
E_{T_i} = \sum_{t \in T_i} \Delta P_t \tag{5}
\]

where \(E_{T_i}\) is the energy prosumption through time-interval \(T_i\), and \(\Delta\) is the time step size. In the TOU example, the total cost of electricity would simply be the energy prosumed in each time-interval (i.e., morning, afternoon, and night) multiplied by the price of each time-interval.

In Section III, we describe and model \(f_c\) for each \(c \in C\) as well as constraints related to pricing tariffs (i.e. \(\xi\)). We discuss constraints related to load models (i.e., \(\zeta\)) in Section IV.

III. DESCRIPTION AND MODELS OF THE COST COMPONENTS

This section describes the cost components that form the building blocks for residential tariffs. Cost components include fixed costs (FC), fixed price of energy (FPE), prosumption-dependent pricing (PDP), critical peak pricing (CPP) and peak demand charges (PDC). Typically, a tariff structure is composed of an FPE and an FC component, where FC includes

<table>
<thead>
<tr>
<th>Cost Component</th>
<th>Known names and variations</th>
<th>Inclusion within literature</th>
</tr>
</thead>
<tbody>
<tr>
<td>FC</td>
<td>Fixed Rate, Flat Rate, Regulated Rate Tariff (RRT)</td>
<td>[22]</td>
</tr>
<tr>
<td>FPE</td>
<td>Time-of-Use (TOU), Time-of-Day</td>
<td>[4], [26], [27], [20], [13], [19], [21], [10], [25], [8], [14]</td>
</tr>
<tr>
<td>CPP</td>
<td>Real-time pricing (RTP)</td>
<td>[26], [27], [15], [11], [22], [17], [23], [16], [25], [12]</td>
</tr>
<tr>
<td>PDP</td>
<td>Variable Peak Price (VPP)</td>
<td>[28]</td>
</tr>
<tr>
<td>PDC</td>
<td>Feed-in tariff (FIT), Standard offer contract</td>
<td>[20], [19], [21]</td>
</tr>
<tr>
<td>PDC</td>
<td>Net metering</td>
<td>[15], [10]</td>
</tr>
<tr>
<td>PDC</td>
<td>Monthly Inclining Block rates (IBR), Lifeline</td>
<td>[29], [30]</td>
</tr>
<tr>
<td>PDC</td>
<td>Daily Inclining Block Rates (IBR)</td>
<td>[11], [11]</td>
</tr>
<tr>
<td>PDC</td>
<td>Quadratic costs</td>
<td>[31], [32]</td>
</tr>
<tr>
<td>PDC</td>
<td>Declining Block Rates (IBR), Decreasing block tariffs</td>
<td>[30]</td>
</tr>
<tr>
<td>PDC</td>
<td>Critical Peak Pricing-Fixed (CPP-F), Critical Peak Pricing (CPP)</td>
<td>[26], [13], [19], [21]</td>
</tr>
<tr>
<td>PDC</td>
<td>Critical Peak Pricing-Variable (CPP-V)</td>
<td>[4], [28]</td>
</tr>
<tr>
<td>PDC</td>
<td>Extreme Day Critical Peak Pricing (ED-CPP)</td>
<td>[33]</td>
</tr>
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</tr>
</tbody>
</table>
interconnection charges and administrative fees, and FPE is a cost that depends solely on the quantity of energy prosumed. The pricing components are summarized in Table II.

A. Fixed Costs (FC)

A utility bill usually includes FC such as interconnection rates, administration charges, local access fees, rate riders, balancing pool allocations, and distribution charges. FC are unaffected by prosumer behaviour and decisions, but may vary by time. For example, the administration charge for interconnection changes by season. Utility bills can consist entirely of FC. For example, in developing countries, the utility may charge consumers FC when the installation of electricity meters is too expensive [30]. Assume the fixed cost for time-interval \( i \) is \( \pi_T \). Thus, we calculate \( f_{FC} \) as follows:

\[
f_{FC} = \sum_{i \in I_{FC}} \pi_T
\]

Note that while \( f_{FC} \) appears in the total cost of electricity, the REMS schedules are not affected by this component.

B. Fixed Price of Energy (FPE)

In FPE, the cost is only dependent on the quantity of energy prosumed. FPE may subdivided into Regulated Rate Tariffs (RRT), Real-Time Pricing (RTP), Time-of-use pricing (TOU), and Feed-In Tariffs (FIT) and net metering, described in Table III-B. Net metering and Feed-In Tariffs may require special modelling, discussed in Sections III-B1 and III-B2.

For \( c = FPE \), the cost of electricity is as follows:

\[
f_{FPE} = \sum_{i \in I_{FPE}} \pi_T E_T
\]

where \( \pi_T \) and \( E_T \) are the electricity price and energy prosumed during time-interval \( T_i \).

1) Net Metering: In net metering, the electricity meter only tracks net energy prosumption, thus electricity is sold to the utility at the same rate that it is bought. Some net metering policies cap net energy sales. In this case (7) is changed to:

\[
f_{FPE} = \sum_{i \in I_{FPE}} \sup \left( f_{T_i}, \pi_T E_T \right)
\]

where \( f_{T_i} \) is the minimum the utility will charge the residential prosumer. Negative quantities signify net sales to the utility.

To adapt (8) to an linear model, it is modified as follows:

\[
f_{FPE} = \sum_{i \in I_{FPE}} f_{T_i}
\]

\[
f_{T_i} \geq f_{T_i}
\]

\[
f_{T_i} \geq \pi_T E_T
\]

where, \( f_c \) is the total cost, and \( f_{T_i} \) is the net limit of electricity sale. The cost at each interval, is the larger between the maximum cost and the value of energy sold to the grid, as shown in (9) - (10).

<table>
<thead>
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<td>Prosumers are charged a fixed rate for all of their energy prosumption and production. For example, this type of electricity pricing is seen in Alberta, Canada [35]. This is the most simple method to implement, and requires any electricity meter.</td>
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<td>Time-of-Use (TOU)</td>
<td>The price of electricity is known, but fluctuates based on the time that energy is prosumed. As a result, the prosumer has the incentive to shift their energy prosumption from high-price energy blocks to low-price energy blocks, which is usually during the night. In reality, the utility typically gives a deterministic price to the prosumers for all hours of a day on the previous day, thus the prices do not necessarily accurately reflect market conditions [26].</td>
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<td>Real-time pricing (RTP)</td>
<td>The price of electricity fluctuates based on supply and demand to reflect the real cost of producing electricity. Since the supply and demand of electricity is non-deterministic, the future price of electricity is unknown, although some retailers could provide deterministic prices for a certain ( t )-ahead, such as in [11]. Thus, in addition to smart metering, an element of forecasting is required in order to make smart energy decisions. In reality, the utility typically gives a deterministic price to the prosumers for all hours of a day on the previous day, thus the prices do not necessarily accurately reflect market conditions [26].</td>
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2) Feed-In Tariff (FIT): In FIT, the electricity consumption may be metered independently from electricity production. For example, in Ontario, Canada, residential electricity consumption and solar power production in a dwelling uses an advance meter that can track both inputs independently. The electricity consumption is on a TOU tariff while solar power production is guaranteed a higher fixed rate. In this case, REMS can independently schedule residential energy production from consumption, and combine the results for the overall prosumption profile.

C. Prosumption-Dependent Prices (PDP)

In PDP, the purchase and sale price of electricity is dependent on the quantity prosumed. For \( c = PDP \), the cost component of electricity is as follows:

\[
f_{PDP} = \sum_{i \in I_{PDP}} \pi_T E_T
\]

where the function \( \pi_T \) is a prosumption-dependent electricity price, which may render \( f_{PDP} \) non-convex or non-linear.

The most common PDP structures are inclining and declining block rates, as described as follows. Figure

1) Inclining Block Rates (IBR): IBR policies are intended to reduce energy prosumption by charging a high price for prosumption over a threshold.

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IBR are used on a monthly energy basis. Prosumers are charged 6.90 cents/kWh for the first 1350 kWh in one month, but pay 10.34 cents/kWh for prosumption beyond 1350 kWh in one month [37]. A 5-tier IBR in California is discussed in [29]. Alternative IBR structures are proposed and formulated in [11] on an hourly basis to flatten daily peak demand.

IBR includes piece-wise constant and non-decreasing price blocks \( b \in B \). We define an IBR Horizon, denoted by \( T_i \), as the interval where energy prosumption is summed. For \( c = PDP \), the IBR cost component is as follows:

\[
f_{PDP} = \sum_{i \in I_{PDP}} \sum_{b \in B} \pi_i \theta_{T_i,b} E_{T_i,b}
\]  

(13)

\[
E_{T_i} = \sum_{b \in B} E_{T_i,b} \quad \forall i \in I_{PDP}
\]  

(14)

\[
0 \leq E_{T_i,b} \leq E_{T_i,b} \quad \forall i \in I_{PDP} \land \sum_{b \in B} E_{T_i,b} \geq 0
\]  

(15)

where \( \pi_i \) is the base price of energy, \( \theta_{T_i,b} \) is the non-negative non-decreasing price multiplier for block \( b \in B \), and \( E_{T_i,b} \) is the energy prosumption in each energy block \( b \in B \) during time-interval \( i \in I_{PDP} \). The maximum energy prosumed in any energy block is \( E_{T_i,b} \). Equation (13) sets the total cost due to PDP by summing costs during each IBR Horizon \( i \in I_{PDP} \). The energy prosumption is binned in (14) into the appropriate energy blocks, with energy block sizes defined in (15)-(16).

In the example above concerning British Columbia electricity prices, \( E_{T_i,1} = 1350 \), \( E_{T_i,2} = \infty \), \( \pi_{T_i,1} = 0.069 \), \( \pi_{T_i,2} = 0.1034 \), and \( t \in T_i \) corresponds to 1 month.

Over a long term (e.g., one month), IBR reduces the overall energy prosumption, while IBR over a short term (e.g., one hour) shifts daily load by shaving peaks [11]. Monthly IBR requires simple metering infrastructure, as we can calculate costs based on a single monthly reading. However, this is not possible with hourly IBR. Further, IBR is often used as a pro-poor policy to offer the less affluent a cheap electricity rate for essential services, while charging more for non-essential loads [30]. Fig. 1 illustrates a three-block IBR, where the marginal price of electricity increases with respect to consumption.

2) Declining Block Rate (DBR): In DBR, each block of electricity costs less than the previous block, such that \( \theta_{T_i,b} \leq \theta_{T_i,b-1} \). This gives a discount for bulk energy prosumption. Since the objective in 1 is to minimize total costs, REMS would have a tendency to fill the energy blocks in ascending order of price. In IBR, the prices of the energy blocks are correctly sorted by its definition; however, for non-increasing prices, such as DBR, it is important to fill the previous energy block before starting on the following one. To achieve this, we use binary variables to force the correct filling order of energy block. For \( c = PDP \), the DBR model is as follows:

\[
f_{PDP} = \sum_{i \in I_{PDP}} \sum_{b \in B} \pi_i \theta_{T_i,b} E_{T_i,b}
\]  

(17)

\[
E_{T_i} = \sum_{b \in B} E_{T_i,b} \quad \forall i \in I_{PDP}
\]  

(18)

\[
0 \leq E_{T_i,b} \leq E_{T_i,b} \quad \forall i \in I_{PDP} \land \sum_{b \in B} E_{T_i,b} \geq 0
\]  

(19)

\[
z_{T_i,b} \leq z_{T_i,b-1} \quad b > 1
\]  

(20)

\[
y_{T_i,b-1}E_{T_i,b} \leq E_{T_i,b} \quad \forall i \in I_{PDP} \land \sum_{b \in B} E_{T_i,b} \geq 0
\]  

(21)

\[
y_{T_i,b-1}E_{T_i,b} \leq E_{T_i,b} \quad \forall i \in I_{PDP} \land \sum_{b \in B} E_{T_i,b} < 0
\]  

(22)

\[
y_{T_i,b} \geq \frac{E_{T_i,b} - E_{T_i,b}}{\delta} + \delta
\]  

(23)

where \( \pi_i \) is the base price of energy, \( \theta_{T_i,b} \) is the non-negative non-increasing price multiplier for each of the blocks \( b \in B \), and \( E_{T_i,b} \) is the energy prosumption in each energy block \( b \in B \) during time-interval \( i \in I_{PDP} \). The maximum energy prosumed in any energy block is \( E_{T_i,b} \). \( z_{T_i,b} \) and \( y_{T_i,b} \) are binary variables representing which blocks are active, and which blocks are fully committed, respectively. \( \delta \) is a very small number used to force the integer value of \( y_{T_i,b} \).

Equation (17) sets the total cost due to PDP by summing costs during each DBR Horizon \( T_i \). The energy prosumption is binned in (18) into appropriate energy blocks. Equation (19) ensures that an energy block is not active until the previous is active. Equation (20)-(21) defines the sizes of each energy block, and indicates if the block \( b \) is active. Equation (22)-(23) allows energy prosumption in the current block \( b \) only if the previous energy block is committed to its maximum value, \( E_{T_i,b} \). \( y_{T_i,b} \) evaluates whether block \( b \) is fully committed. Applying (17)-(24) instead of (13)-(16) to IBR is possible, however it would create unnecessary binary variables \( z_{T_i,b} \) and \( y_{T_i,b} \).

Fig. 1 illustrates DBR, where the marginal price of electricity decreases with respect to consumption.

D. Critical Peak Pricing (CPP)

On 10-15 days per year, called critical days, the cost of producing electricity is unusually high, and the utility charges a higher rate for a predefined period. Prosumers are alerted a day ahead to manage their electricity demand accordingly. CPP is paired with FPE to create a viable tariff. Several CPP structures exist, such as CPP-Fixed (CPP-F), CPP-Variable (CPP-V), Peak time rebates (PTR), Extreme Day Pricing (EDP) and Extreme Day-CPP (ED-CPP), shown in Table III-D. For \( c = CPP \), the cost is modelled as follows:
TABLE III
VARIATIONS OF CRITICAL PEAK PRICING (CPP)

<table>
<thead>
<tr>
<th>Tariff</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Critical Peak Pricing-Fixed (CPP-F)</td>
<td>Normally: TOU Pricing. Critical days: TOU+CPP with fixed CPP price during the predefined constant peak demand period. A reference to CPP usually implies this model.</td>
</tr>
<tr>
<td>CPP Variable (CPP-V)</td>
<td>Difference from CPP-F: The peak demand period may vary its time of day and duration.</td>
</tr>
<tr>
<td>Extreme Day CPP (ED-CPP)</td>
<td>Difference from CPP-F: Flat rate pricing on normal days.</td>
</tr>
<tr>
<td>Extreme Day CPP (EDP)</td>
<td>Difference from CPP-F: On critical days, CPP rate is charged for the entire day.</td>
</tr>
<tr>
<td>Peak Time Rebate (PTR)</td>
<td>This model offers a different perspective by providing a rebate on critical days for reduced demand.</td>
</tr>
</tbody>
</table>

where \( \pi_t \) is the price, \( \alpha_{T_i} \) is the CPP state in period \( i \in I_{CPP} \), \( E_{T_i} \) is the energy prosumed in period \( i \in I_{CPP} \), and \( E^R_{T_i} \) is the reference energy prosumption for the PTR tariff.

The cost of electricity is calculated in (25). Since \( \alpha_{T_i} \) is usually known one day in advance, it would be deterministic for the first 24-hours and stochastic afterwards. However, since most REMS have a scheduling horizon of 24 hours or less, \( \alpha_{T_i} \) is generally deterministic.

E. Peak Demand Charges (PDC)

If the cost of electricity is dependent on the maximum prosumption, then a PDC cost component exists. For example, PDC may be based on the maximum 15-minute average demand that occurred throughout the entire month, or it could be charged daily, based on the maximum 1-hour average power that occurred during a peak period (e.g., 5-8 PM) [19]. The cost component for \( c = \text{PDC} \) is as follows:

\[
f_{\text{PDC}} = \sum_{i \in I_{\text{PDC}}} \pi_{T_i} \alpha_{T_i} \left[ E_{T_i} - E^R_{T_i} \right] \tag{25}
\]

where \( \pi_{T_i} \) is the PDC price, \( \langle P^{(k)}_t \rangle \) is the moving average function, \( P_t \) is the demand at time \( t \), \( k \) is the number of time steps in the averaging period, \( \pi_{T_i} = 0 \) when PDC is not active for period \( i \). We linearize (26) in the following:

\[
\langle P^{(k)}_t \rangle = \frac{1}{k} \sum_{i=t-k+1}^{t} P_i \quad \forall t \in T \tag{27}
\]

\[
\text{Peak}_{T_i} \geq \langle P^{(k)}_t \rangle \quad \forall t \in T, \forall i \in I_{\text{PDC}} \tag{28}
\]

\[
\text{Peak}_{T_i} \geq 0 \quad \forall i \in I_{\text{PDC}} \tag{29}
\]

\[
f_{\text{PDC}} = \sum_{i \in I_{\text{PDC}}} \pi_{T_i} \text{Peak}_{T_i} \tag{30}
\]

where \( \langle P^{(k)}_t \rangle \) is the moving average prosumption between time \( t - k + 1 \) and \( t \) and \( \text{Peak}_{T_i} \) is the maximum \( k \)-average prosumption in time-interval \( i \). The moving average prosumption in (27) ensures that instantaneous power spikes are not counted towards the PDC. The peak power demand is the larger of the maximum moving average prosumption for the PDC horizon in (28), and 0 in (29). Equation (30) calculates the cost of the PDC based on the maximum prosumption, \( \text{Peak}_{T_i} \).

PDC programs are often paired with TOU pricing, to capture a user’s contribution to the system’s peak demand with minimal additional metering infrastructure. However, it is not necessarily effective as the user’s peak demand may not coincide with the system peak demand. In addition, if PDC is evaluated over a long term (e.g., one month), one day of high prosumption could result in low incentive to reduce demand for the rest of the evaluation period.

IV. RESIDENTIAL LOAD MODELS

In this section, we discuss the constraints for various load devices in the REMS optimization model, i.e., \( \zeta \) in (1). In a typical dwelling, there may exist several load devices, such as lights, dishwashers, and electric vehicles. Each residential load device has unique characteristics, which makes modelling complex. Some of REMS formulations in the literature model each device separately, which is complex, but accurate, whereas some model them with less detail. For example, in [10], [13], a water heater, space heater, electric vehicle, pool pump, photovoltaic system and must-run services are modelled separately. Conversely, all loads in [11] are modelled uniformly. Thus, it is important to evaluate the trade-off between simplicity and accuracy depending on modelling requirements.

To address this trade-off in the present research, we propose to partition load devices into a limited number of response classes, such that device formulations can be specific to some degree without being excessive. Several works use a similar approach. For example, in [18], responses are classed into baseline, regular, and burst loads. The response classes in our model includes Uncontrollable Loads (UL), Price Responsive Loads (PRL) , Burst Loads (BL) and Regulating Loads (RL), as summarized in Table IV, and further described in subsequent sections. This approach for partitioning the loads in optimization-based models has the advantages of providing relatively accurate load models, providing flexibility in selecting a specific load model for each device, and allowing more response classes to be created as new devices with unique properties that cannot reasonably fit into any of the response classes are added to the home area network.

The proposed PRL model allows for continuous or step decreases in consumption as a function of pricing. The proposed BL model allows separating a device operation into multiple time segments, such as separating the timing of the wash and rinse cycles from the clothes washer. The proposed RL model allows for energy depletion due to non-grid energy usage, such as hot water usage in a hot water tank.

Let \( r \) denote a response class, where \( r \in R = \{ UL, PRL, BL, RL \} \). Also, let \( D_r \) refer to the set of all devices, and \( D_r \) represent the devices in response class \( r \). In the subsequent sections, we provide linear models for each...
response class and further explain how a typical device may fit in a certain response class.

A. Uncontrollable Loads (UL)

Uncontrollable loads prosume power independently of action taken by the REMS. This includes electrical losses in the dwellings, power required to run the power meters and REMS, and any device controlled solely by the prosumer. For \( d \in D_{UL} \), uncontrollable loads are modelled as follows:

\[
P_{t,d} = \ell_{t,d} \quad \forall t \in T
\]  

(31)

where \( \ell_{t,d} \) is the uncontrollable prosumption at time \( t \).

B. Price-Responsive Loads (PRL)

The prosumption of PRL are deterministic with respect to the price of electricity. PRL can be aggregated in order to form a comprehensive PRL curve. There are two interpretations to using price-responsive loads in residential energy simulations. In the first interpretation, the prosumer changes their prosumption patterns in response to prices (e.g., decision to not watch television). Although it is often assumed that electricity is inelastic, studies show that prosumers do change their behaviour due to prices [4], [26], and may have an improved ability to respond to prices with better control mechanisms, such as using smart phones to control loads [38]. In the second interpretation, REMS throttle prosumption based on the price of electricity (e.g., REMS dimming lights). The price of electricity may dictate to the REMS, the quantity of energy services to curtail. Thus, prosumption in this case is a function of electricity price, as follows:

\[
P_{t,d} = p_{t,d}(\pi_t) \quad \forall t \in T
\]  

(32)

where the function \( p_{t,d} \) calculates the prosumption based on price, denoted by \( \pi_t \).

If the electricity price is constant at time \( t \) (i.e., not variable due to PDP nor PDC), then the function \( p_{t,d} \) can be arbitrary without affecting the integrity of the model. Otherwise, \( p_{t,d} \) needs a specific formulation. Assuming that the price of electricity is the price of energy plus CPP if it is active, that is, \( \pi_t = \sum_i \pi_{t,i} \) for \( t \in T, i \in I_{FPE} \cup I_{CPP} \), we propose two methods for formulating PRL. In both formulations, \( P_{t,d} \) is the prosumption of PRL, \( \pi_{t,d,pb} \) is the price threshold, \( \pi_{t,d,pb} \) is the price of electricity in each price block, and \( \pi_t \) is the price of electricity.

1) Gradual PRL: The first formulation, similar to the IBR formulation, allows for a gradual decrease in prosumption due to the increase in price. This may be useful for price responsive dimming of lights, or for simulating aggregate elasticities of demand. For example, it is possible to create a piece-wise linear function simulating the elasticity of demand of 0.2 in [27]. For \( d \in D_{PRL} \), gradual PRL are modelled as follows:

\[
P_{t,d} = \pi_t - \sum_{pb \in PB} \kappa_{t,d,pb} \pi_{t,d,pb}
\]  

(33)

\[
0 \leq \pi_{t,d,pb} \leq \pi_t
\]  

(34)

\[
\pi_t = \sum_{pb \in PB} \pi_{t,d,pb}
\]  

(35)

where \( \pi_{t,d} \) is the prosumption, if electricity were free, and \( \kappa_{t,d,pb} \) is a response constant. In (33)-(35), \( \kappa_{t,d,pb} \) and \( \pi_{t,d} \) are non-negative and \( \kappa_{t,d,pb} \leq \kappa_{t,d,pb-1} \). Figure 2 illustrates consumption as a function of price.

2) Step PRL: The second formulation models a downward step function, in order to mimic individual devices being turned off when the price of electricity increases past a threshold. For \( d \in D_{PRL} \), one can write:

\[
P_{t,d} = \sum_{pb \in PB} \kappa_{t,d,pb} v_{t,d,pb}
\]  

(36)

\[
v_{t,d,pb} \geq \delta [\pi_{t,d,pb} - \pi_t]
\]  

(37)

\[
v_{t,d,pb} \leq 1 + \delta [\pi_{t,d,pb} - \pi_t]
\]  

(38)

where \( v_{t,d,pb} \) is the incremental load associated to a price block, \( \delta \) is a small number that forces the binary variable \( v_{t,d,pb} \) to one value, and \( \phi_{t,d,pb} \) is the response constant. In (36), \( \phi_{t,d,pb} \) is positive.

C. Burst Loads (BL)

The operations of some devices require a specified power draw for a specified quantity of time. Devices such as washing machines, drying machines and dishwashers, are called burst loads in [18]. Other names for BL include service loads, scheduled loads, and time-sensitive loads. Due to the binary nature of these loads, they need to be scheduled using mixed-integer programming. Most of the literature assumes BL are

<table>
<thead>
<tr>
<th>r</th>
<th>Model Name</th>
<th>Description</th>
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<tbody>
<tr>
<td>UL</td>
<td>Uncontrollable loads</td>
<td>Prosumption that cannot be influenced by the REMS. Example: Power required to run electricity meter.</td>
</tr>
<tr>
<td>PRL</td>
<td>Price-responsive loads</td>
<td>Prosumption that responds solely to the current price of energy. Example: Light dimming.</td>
</tr>
<tr>
<td>BL</td>
<td>Burst loads</td>
<td>Devices with a predefined operation schedule with flexibility on the start time. Example: Washing machines.</td>
</tr>
<tr>
<td>RL</td>
<td>Regular loads</td>
<td>Devices that must regulate energy states, and act as an energy store. Example: Freezer.</td>
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uninterruptible, and run a constant power draw from the beginning to the end of the operational period. We propose the time-series formulation below to allow for multiple-uses within specified intervals, which could represent segmenting the clothes washer load’s wash and rinse cycle, as follows:

\[
P_{t,d} = w_{t,d} P_d + [1 - w_{t,d}] P_d \quad \forall t \in T \tag{39}
\]

\[
w_{t+1,d} - w_{t,d} = w_{t+1,d} - w_{t,d} \quad \forall t \in T \tag{40}
\]

\[
w_{t+1,d} \leq w_{t,d} \quad \forall t \in T \tag{41}
\]

\[
w_{t,d} = w_{t+1,d} \quad \forall t \in T \tag{42}
\]

\[
I_d w_{t,d}^+ = \sum_{t=I_d}^{t+I_d} w_{t,d} \quad \forall t \in T \tag{43}
\]

\[
w_j^+ \leq \sum_{t=I_j}^{T_j} w_{t,d}^+ \quad \forall j \in J_d \tag{44}
\]

where \(d \in D_{BL}\). Also, \(P_{t,d}, P_d\), and \(P_d\) are the power, maximum power, and stand-by power prosumption, \(w_{t,d}, w_{t,d}^+, \) and \(w_{t,d}^-\) are the device statuses indicating operation, start-up, and shut-down, respectively. \(I_d\) is the operational interval of device \(d\), such that once the device has run for \(I_d\) periods, \(L_j\) and \(T_j\) are the intervals which allow the device \(d\) to start, \(w_j^+\) and \(w_j^-\) are the number of times that the device can start its operation in each starting interval. Equation (39) links prosumption to the operation of the device. The necessary relations to track the beginning and the end of a scheduled operation are in (40)-(42). In (42), the device operates for the operational interval once started. In (43), the device operates between \(w_j^+\) and \(w_j^-\) times in the interval \(t = [L_j, T_j]\), to ensure that the consumer is not overly inconvenienced due to the delay in load operations. A load with multiple cycles can be interrupted into multiple segments in (39) - (44) into \(w_j^+\) or \(w_j^-\) segments of \(I_d\) periods, such as in [19]. Alternative burst load formulations are available in [11], [15], [16], [24], including a heuristic model in [6].

D. Regulating Loads (RL)

Regulating Loads, such as fridges and space heaters, have to regulate a device’s energy state (e.g., humidity, heat, pressure...) in the proximity of an ideal set-point by controlling its prosumption. RL share many characteristics with energy storage units, despite some devices being unable to discharge power. RL models are found in [7], [24], [39], [40], [41], [42], [17], [22]. This paper uses a formulation generally consistent with the literature. For \(d \in D_{RL}\), we propose the following linear formulation for RL response class:

\[
P_{t,d} = P_{t,d}^+ - P_{t,d}^- \quad \forall t \in T \tag{45}
\]

\[
0 \leq P_{t,d}^+ \leq a_{t,d} P_d \quad \forall t \in T \tag{46}
\]

\[
0 \leq P_{t,d}^- \leq a_{t,d} P_d \quad \forall t \in T \tag{47}
\]

\[
E_{t+1,d} = E_{t,d} - U_{t,d}^+ \tag{48}
\]

\[
\beta_d \left[ E_{t,d}^A - E_{t,d} \right] \tag{49}
\]

\[
E_{t,d} \leq E_d \quad \forall t \in T \tag{50}
\]

where \(E_{t,d}, E_d\), \(E_{t,d}^A\), \(E_{t,d}^-\), are the device energy state’s scheduled, minimum, maximum, scheduled usage. \(E_{t,d}^+\) and \(E_{t,d}^-\), \(E_{t,d}^A\) are the scheduled, minimum and maximum, non-grid energy depletion respectively. \(E_{t,d}^A\) are the forecasted external/ambient conditions, respectively, while \(a_{t,d}\) represents the device’s availability to be controlled. \(P_{t,d}, P_{t,d}^+, \) and \(P_{t,d}^-\) are the net prosumption, power input into device \(d\), and output from device \(d\), respectively. \(P_{t,d}, P_{t,d}^+\), and \(P_{t,d}^-\) are the minimum and maximum prosumption, respectively. \(\eta_d, \tau_d\), and \(\beta_d\) are parameters of the devices representing the charging efficiency, and self-discharge rate constants, respectively, where \(\beta_d = (1 - e^{-\Delta/\tau_d})\).

A device’s overall prosumption is tracked in (45), where \(P_{t,d}^+\) designates power going from the grid towards the devices, before efficiency losses. The limits of grid input and output power from devices subject to its availability, \(a_{t,d}\), is set in (46)-(47). For example, \(a_{t,d} = 1\) when a device has a severe grid failure or a grid failure. \(P_{t,d}^-\) otherwise. If a device cannot provide electricity back to the grid, the maximum output power, \(P_{t,d}^- = 0\). The inter-temporal energy state is based on Newton’s law of cooling (first order), and can be explained in three terms in (48): the first part affects the next state by accounting for the previous state and non-grid energy depletion, the second part measures state change due to grid power prosumption and production, and the third part measures self-discharge. The acceptable energy state limits are in (49). The limits of non-grid stock depletion, from device \(d\), are in (50). Equations (49)-(50) ensure that energy levels remain within reasonable limits to maintain a high quality of life for the consumer.

When \(E_{t,d}^- > E_{t,d}^+\), then REMS have some control and scheduling flexibility concerning how the energy is depleted. For example, \(E_{t,d}^-\) may include the quantity of energy used from a plug-in hybrid electric vehicle’s battery, when it may need to decide between using the vehicle’s battery or fossil fuels. When \(E_{t,d}^+ = E_{t,d}^-\), then the depletion cannot be controlled. For example, this may include energy loss when opening the door of a fridge. Finally, when \(E_{t,d}^+\) is negative, it implies that an external factor is contributing to the state increase, such as the heat from a person or stove usage that increases the heat state in a dwelling.

V. Conclusion

In this paper, we presented a modelling framework for creating electricity tariff structures and residential load, by partitioning the tariff into cost components, and by binning electrical devices operations into response classes for limiting modelling requirements. This framework can be used to evaluate the behaviour and value of REMS under various electricity tariffs. Extensions of this work can include modelling considerations for consumer preference and comfort, as well as emissions related to the usage of devices.

We presented six electricity cost components, which can be combined in various manners to form various tariff structures. To the knowledge of the author, this is the first work that classifies and discusses tariff structures in the specific modelling context of modelling energy prosumption behaviour when using a REMS. Additionally, this paper discusses certain
implications of tariff structures on REMS and the metering infrastructure requirement. A natural extension of this work is to include costs associated to the scheduled operations of devices (e.g. the wear on equipment, and the cost of non-electric sources of energy).

The presented load model allows for different types of devices to be modelled with reasonable accuracy while limiting the uniqueness and complexity of each device. This facilitates the ability for the REMS to add and remove devices from its formulation with relative simplicity. The load types modelled are uncontrollable loads, price-responsive, regulating loads, and autonomous energy sources.

Implementing the proposed framework and investigating the outcomes for typical residential consumers and discussing how policy-makers could benefit from the insights provided by this framework are the subjects of the authors’ future work.

REFERENCES