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RTCM 3.0 Implementation in Network RTK and Performance Analysis

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by

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Abstract

This thesis discusses the RTCM 3.0 implementation in network RTK positioning using the Department of Geomatics Engineering PLAN Group's network software, MultiRef^{TM}, and addresses novel features of the new standard. Post-mission tests have been performed with field data from the Southern Alberta Network (SAN). Three interpolation techniques are discussed, namely the distance-weighted, plane and collocation methods.

Results show that the RTCM 3.0 approach is a preferable implementation for network RTK positioning by reducing the sizes of the network RTK corrections. All three interpolation techniques are effective and obtain similar results in the position domain when the network ambiguities are properly resolved. However, when the network software is incapable of resolving many of the network ambiguities, the network approach does not show significant improvement over the single baseline approach when *Kinematic Ambiguity Resolution* (KAR) is enabled. When KAR is disabled, i.e. positions are computed using the IF mode, the network approach is marginally better than the single baseline approach.

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List of Symbols, Abbreviations and Nomenclature

Symbol	Definition
ρ	Range between SV (at transmit time) and receiver (at receive time)
I	Ionospheric delay
f_1, f_2	Carrier signal frequency (1575.42 MHz for L1, 1227.60 MHz for L2)
T	Measurement delay due to troposphere
m	Measurement delay due to multipath
c	Speed of light
δt_{rec}	Receiver clock error
δt_{sv}	Satellite clock error
υ	Measurement noise
λ	Wavelength of L1 or L2 carrier
δr_{sv}	Orbit error
δp_{rec}	The error of receiver positions relative to the true positions
N	Integer carrier-phase cycle ambiguity
S'	Signal at the observation points
S	Signal at the prediction points
n	Observation noise
r^*	Residual
C	Covariance matrix
δp_{sv}	Orbit error
$\bar{\phi}$	Measurement-minus-range observable
T'	Residual tropospheric error
D	Single difference matrix
μ	Elevation mapping factor
ε	Elevation
p	Position vector

Abbreviations and Acronyms

AR	Ambiguity Resolution
bps	Bits per second
C/A	Coarse/Acquisition
CD	Correction Difference
CRC	Cyclic Redundancy Check
DD	Double Difference
DDC	Double Difference Corrections
FKP	(German) Fl¨achen Korrektur Parameter, area correction parameter
GCPCD	Geometric Carrier Phase Correction Difference
GF	Geometry Free
GNSS	Global Navigation Satellite System
GPS	Global Positioning System
ICPCD	Ionospheric Carrier Phase Correction Difference
IF	Ionosphere Free
KAR	Kinematic Ambiguity Resolution
MPC	Modulated Precision Clock
NMEA	National Marine Electronics Association
ppm	Parts Per Million
PRC	Pseudorange Corrections
PRN	Pseudo-Random Noise code
RINEX	Receiver Independent Exchange
RMS	Root Mean Squared
RTCM	Radio Technical Commission for Maritime
RTK	Real-Time Kinematic, implies carrier phase based
SAN	Southern Alberta Network
TCP/IP	Transmission Control Protocol/Internet Protocol
UDP	User Datagram Protocol
UTC	Coordinated Universal Time
VRS	Virtual Reference Station

Chapter One: Introduction

Over the years, the Global Positioning System (GPS) has evolved into a significant tool to meet civilian navigation and positioning requirements worldwide. In order to achieve centimetre or even millimetre level accuracies, the double-differenced (DD) GPS technique must be utilized by forming double-differenced carrier phase observables and resolving the DD integer carrier phase ambiguities. However, this real-time kinematic (RTK) positioning application is limited by differential ionospheric errors, differential tropospheric errors, differential satellite orbital errors and multipath. During periods of extremely high ionospheric activity, the maximum distance from the reference station might be as low as a few to 10 km (Lachapelle 2000). Unlike single reference station RTK approaches where positioning accuracy decreases while the baseline increases, network RTK approaches ideally provide positioning with errors independent of the rover position within the network. Moreover, by integrating and optimizing the information from multiple reference stations, network RTK covers the desired area with fewer reference stations compared with the single reference station approach (Raquet 1998).

In both the single reference station and network RTK approaches, corrections must be transmitted to the rover receiver. In order to standardize the format and content of the corrections, the Special Committee 104 of the Radio Technical Commission for Maritime services (RTCM SC-104) has developed a standard for Differential Global Navigation Satellite System (GNSS) Service, referred to as RTCM Version 2 (RTCM 2001). However, since this Version 2 standard does not readily support network RTK, the

Committee is currently developing a new standard - RTCM 3.0 (RTCM 2004a), which will address some of the problems associated with application of Version 2, and which will readily accommodate network correction messages. A detailed description of how RTCM 3.0 is implemented in network RTK positioning design and operation is given in this thesis and also an investigation of network RTK positioning performance under different scenarios is presented.

1.1 Background & Motivation

Chief among the systematic errors affecting the RTK rover performance are multipath, atmospheric and orbital errors. Differential atmospheric and orbital errors will increase with an increase of baseline length. Therefore, the single reference station RTK approach is limited with respect to the distance between reference and rover receivers, i.e. the baseline length. Generally the positioning accuracy will decrease with an increase of the baseline length. However, the network RTK approach offers the possibility of homogeneous positioning accuracy within a network, with the use of fewer stations to cover the desired area. Fortes *et al* (2000) show that the network approach demonstrates improvements in all of the observation, position and ambiguity domains and, therefore, network RTK positioning is a strong candidate as the preferred solution for large area high-precision satellite positioning applications.

Networks of reference stations have been installed in several countries (Townsend *et al* 2000), such as Sweden's SWEPOS (Jonsson *et al* 2003), the Brazilian Network for

Continuous Monitoring of GPS (RBMC) (Fortes *et al* 1998), Japan's Geographical Survey Institute (GSI) GPS network (Petrovski *et al* 2000) and the Southern Alberta Network (SAN) (Alves *et al* 2004b) which is maintained by the Position, Location And Navigation (PLAN) Group of the University of Calgary. However, because the RTCM Version 2 does not readily support network RTK approaches, the data exchange in these installations is still based on proprietary information messages such as the Flächen Korrektur Parameter (FKP) implementation (Wübbena *et al* 2001, 2002) or complex and expensive two-way data links such as the virtual reference station (VRS) implementation (Vollath *et al* 2000a, Landau 2003, Alves 2004a). The way in which VRS and FKP are currently implemented according to the Version 2 standard is discussed in Section 1.2. The need for a standardized network information exchange is understood by researchers and industry. Development of a standardized network information protocol should help to overcome the disadvantages of the above two network RTK implementations. Moreover, in practice, Version 2 has been the subject of an increasing number of complaints regarding parity overhead, awkward data format and an insufficient degree of integrity for safety-of-life applications. The increasing complaints and new demands impelled the RTCM SC104 to develop a new standard, RTCM 3.0.

Within Subcommittee SC104 of the RTCM, a working group headed by Dr. Hans-Jürgen Euler was established to define a set of appropriate network message types by forming the "GNSS Network Corrections" Group within the forthcoming RTCM 3.0 standard (Kalafus & Van Dierendonck 2003). However, steady progress has been slow due to the complexity of the issues involved. Proper interoperability between different manufacturers' equipments must be performed so that the standard can consolidate different opinions and present a model that all manufacturers would agree upon. In 2004, some off-line interoperability tests were performed in conjunction with several major receiver manufacturers. These tests verified that interpretation of the draft standard was unambiguous and consistent among all participants Moreover, a real-time interoperability test was carried out on November 22, 2005 at SAPOS, a GPS network covering the Germany. Final results show that the deviations between different solutions with different services are within the typical range of variation (RTCM 2005).

By standardizing network information and processing models, not only the size of network RTK corrections but also the satellite-independent error information can be reduced (Euler *et al* 2001, Euler & Zebhauser 2003). Proposed by Euler *et al* (2001), a simplified approach known as the "Master-Auxiliary Concept" where network corrections are defined relative to the master station, is now incorporated into the draft of Version 3 Proposed Messages (RTCM 2004b). With the standardization of network messages, some of the problems, such as bi-directional data link or modeled information transmission, of the current network RTK implementation method could be overcome.

MultiRef™ is a network software developed by the PLAN Group of the University of Calgary. Following the VRS implementation method, MultiRef™ estimates the network ambiguities and creates a VRS to be transmitted to the rover receiver. However, to surmount the shortcomings of VRS implementation, in respect to two-way communication, MultiRef™ is broken down into two parts; one part installed at the control centre to resolve ambiguities, and the other part installed in a laptop or PC which communicates with the rover receivers to get approximate positions of the rover receivers and generate a virtual reference station for each rover. The communication between these two parts relies on customized network messages defined by the PLAN Group. A detailed description of MultiRef^{TM} is given in Chapter Four. As a standard readily supporting GNSS network RTK, RTCM 3.0 is yet to be formally promulgated. RTCM SC104 expects that the standardization of the network RTK information could eliminate the occurrence of multifarious proprietary network messages. To implement RTCM 3.0, MultiRef^{TM} has modified accordingly. A detailed description of the modification is given in Chapter Four.

1.2 Literature review

Currently there are two popular GPS network RTK implementation methods - namely VRS and FKP.

VRS implementation, which was first proposed by Vollath *et al* (2000a), consists of three main steps (Lachapelle & Alves 2002): first, the network software estimates the double-differenced ambiguities and further estimates the corrections for each station and each satellite; second, the corrections are interpolated to the position of the rover; third, the corrections are transmitted to the rover receiver by generation of a virtual reference station in a format of single reference station RTK messages defined in RTCM Version 2. The main advantage of VRS is that nothing needs to be modified at the rover receiver end.

However, a concomitant defect of this advantage is that the rover receiver is completely unaware of the existence of the network. This will possibly sacrifice the flexibility of the rover receivers, which may lose the option to optimize the information on their own. Moreover, since the VRS approach is only effective in a limited area, the network service either needs to generate very dense VRSs to cover the desired area or relies on a two-way communication link. The first method requires significant resources to broadcast the corrections to all the VRSs. Network service providers usually prefer the second method whenever a two-way communication link is possible. In the second method, the rovers not only receive VRS information, but also additionally transmit, via National Marine Electronics Association (NMEA) messages, their approximate positions to the control centre where the network software is installed. Whereas, VRS is still a good concept to transit from single reference approach to network approach, since the rover receivers fit into the network right away. Vollath *et al* (2000b) show that, by employing the VRS concept, network RTK could provide performance at the accuracy level comparable to the short single baseline approach when the rover receivers are located up to 40 km or more from the nearest reference station.

One benefit of the FKP method is in simplifying the VRS concept, which involves reducing the three previous steps into two. The first step is the same. In the second step, instead of building a virtual reference station, a set of coefficients modeling ionospheric, tropospheric and orbital effects are calculated for each satellite to cover a specific network area at specific time intervals (at least every 10 s) (Euler *et al* 2001). The coefficients are then transmitted to the rovers, which then attempt to use these coefficients as a basis for generating their own corrections. The major disadvantage, however, is that it utilizes a customized RTCM Type 59 proprietary message (Wübbena & Bagge 2002). Another disadvantage is that it is not fully compliant with the RTCM standard in that the standard clearly states that the reference station data should not be modified to correct for any atmospheric or orbital error while it is inherent in the FKP concept. Some investigators argue that this poses a risk of inconsistent tropospheric modeling between server and rover (Landau *et al* 2003).

Another new method worthy of mention, referred to as the in-receiver method, was proposed by Pugliano *et al* (2003) and Alves *et al* (2004c). In this approach, no control centre is needed since all of the required information is collected and processed at the rover receiver. That is why the method is named in-receiver or tightly coupled. The single baseline RTK positioning and network adjustment are integrated into a tightly coupled filtering routine, i.e. optimizing the final positioning accuracy by synthesizing all of the information from the reference stations and rovers. The results of Alves *et al* (2004c) show that the in-receiver algorithm can improve 3D positioning accuracy by about 20% over the correction-based network approach. However, although this method purports to be effective, practically speaking, either the processing power requirements or the simplicity requirements of the receiver do not allow the rover receiver to perform complex network adjustments at this time. However, this may not be a limitation in the future with the rapid evolution of low power, low cost processors. An alternative is to transmit the rover's measurements to the control centre. However, this also raises the argument of employing a two-way communication link.

Regardless of whether VRS, FKP or in-receiver implementation is employed, due to their common root in the latest RTCM Version 2, they all represent attempts to produce network RTK capability based on a standard that was not designed for this purpose. However, Version 2, which was initiated more than 10 years ago, is subject to increasing complaints over both its burdensome parity scheme and out-of-date data format (Kalafus & Van Dierendonck 2003), among other concerns. Moreover, the rapidly emerging developments in GNSS differential systems call for a new standard to readily accommodate new signals and frequencies, emerging GNSS systems such as Galileo and new applications.

The new Version 3 standard has been designed to incorporate these new requirements and to avoid the problems associated with Version 2, as cited above. As to the content of the network correction messages, three proposals have been discussed in the RTCM SC104 Working Group as elements of the RTCM 3.0 standard.

The first proposal was published by Townsend *et al* (2000). Based on a grid model, this proposal segmented the network area into a grid of a certain size. Two types of messages are defined: “Grid Definition Message” and “Correction Message”. The grid definition message defines the network region, grid size and grid ID assignment. The correction message contains corrections for each satellite in each specific grid. This proposal is sub-optimal because very important information, namely height, is missing in the grid definition message (Euler *et al* 2002).

The second proposal, known as the Master-Auxiliary Concept, was drafted by Euler *et al* (2001). By designating one station in the network as being the master station, this proposal uses a between-station single difference approach of RTCM correction messages previously defined in RTCM Version 2. By the single differencing of corrections, this proposal achieves maximal reduction in the bulk of the original data content from the reference station observations. First, the legacy standard RTCM 2.3 is followed to remove the geometry range in the GNSS observation data by formulating the correction messages (types 20 and 21). Secondly, between-station single difference corrections are formed by subtracting the common master station variations in the ionospheric, tropospheric, and orbital effects. Lastly, if the network ambiguities are successfully determined, these ambiguities can be further eliminated from the corrections. Thus, the remaining components of the corrections only contain the ionospheric, tropospheric, and orbital affects under the assumption of correctly fixed network ambiguities. Related deductions are shown in the Chapter Three. As validated by Euler *et al* (2002), the range of the correction can be as small as $\pm$24 m, which very effectively capsulate the network information, as compared with the original observation set.

The High-Low rate satellite concept is presented in the third RTCM proposal (RTCM 2002). The basis of the High-Low rate concept and the Master-Auxiliary concept are almost identical, except for the distinction made in the High-Low rate concept between high and low rate satellites. The information associated with high-rate satellite changes more rapidly than the information of the other, so-called low-rate satellites. By distinguishing high and low rate satellites, further reduction of size of corrections needed to be transmitted can be achieved, however at a cost of an increasing of complexity of the network software.

Finally, the Master-Auxiliary concept was accepted in the RTCM 3.0 SC104 committee for use in network RTK messages. Hence, in the RTCM 3.0 scheme, the task of network software is simplified considerably, leaving only the first step – "resolving ambiguities". The network corrections are transmitted to the rover and the rover is responsible for interpolation and application of the corrections. Details are given in Chapter Four.

1.3 Objectives and Tasks

RTCM 3.0 standardizes network information and processing models. It puts an end to multifarious proprietary network messages, and also provides more freedom to the network rover receivers than older network implementation methods. Given that RTCM 3.0 clearly offers advantages over these older implementation methods, this thesis has the following objectives:

1) *To evaluate several GNSS Network Correction message structures in terms of accuracy, versatility, and transmission efficiency.* Initiated by Euler *et al* 2001, these messages have been thoroughly discussed in the literature in a theoretical sense; however, more practical network tests need to be performed by various network service providers to evaluate the feasibility and effectiveness of RTCM 3.0.

2) *To assess various interpolation methods that could be implemented at the rover end to interpolate the network corrections onto the position of the rover.* The RTCM 3.0 scheme transmits only network corrections to rovers, thus leaving it to the rovers to determine how best to interpret and apply the corrections, according to a rationale contained in decision rules embedded in the rover software. After the corrections are applied, the errors between the rover and master reference station will be reduced. The remaining errors will affect ambiguity resolution and position accuracy at the rover end. Hence, an assessment of the performance of various interpolation methods is of great significance in the implementation of RTCM 3.0 in network RTK.

3) *To evaluate network performance under different ionospheric conditions.* The key task of network software is the successful resolution of ambiguities across the network. However, high ionospheric effects will frustrate this assumption and, consequently, reduce the effectiveness of the network software. Compulsorily using network corrections generated from float ambiguities may result in unpredictable outcomes in the measurement domain, thus worsening the network solution. The evaluation methodology contained herein assists the rover in selecting the correct occasions on when to switch from a network solution to a single reference station solution.

MultiRef^{TM} network software will be used to develop and test the means of reaching these objectives. The following three tasks are arranged to achieve the objectives described above.

1) *Implement the RTCM 3.0 in MultiRef^{TM}*. In the previous MultiRef^{TM} infrastructure, some components were developed to interpolate the network corrections and generate the VRS. A new component called the RTCM 3.0 engine is developed by the author using C++ language. It replaces these earlier components and generates RTCM 3.0 Network RTK messages to be transmitted to the rover receivers. In the meantime, the raw observations of the master station will be transmitted to the rover through RTCM 3.0 GNSS RTK messages.

2) *Develop the related components to make baseline processing software compatible with RTCM 3.0.* RTCM 3.0 is not currently supported by any commercial baseline processing software; thus, some components are developed to interpolate the network corrections onto the rover's position and apply the rover correction difference back onto the Master station raw observations. Therefore, the newly corrected observations will form the input of commercial software. In this task, three interpolation methods are proposed: (i) distance weighted, (ii) plane and (iii) collocation interpolation.

3) *Test and evaluate the performance of the RTCM 3.0 format under various ionospheric effects.* Medium and high ionospheric scenarios will be considered and

their effects on the resulting network RTK solution will be investigated and compared with the single reference station approach.

1.4 Outline

In Chapter 2, the structures of RTCM Versions 2 and 3 are reviewed. Besides network RTK, the new standard readily accommodates the new GNSS system (Galileo), as well as the new GPS signals (i.e. L2C, L5). The advantages of the new data format and parity scheme of RTCM 3.0 are evidenced in high message integrity and efficiency, as compared with RTCM Version 2.

Chapter 3 introduces the key concepts of network RTK messages defined in RTCM 3.0: Ambiguity level, Master-Auxiliary, Dispersive-Non-dispersive and subnetwork definition. This full set of concepts aims at a reduction of the sizes of the network RTK corrections and the transmission of satellite-independent error information.

Chapter 4 describes how RTCM 3.0 is implemented in the network software MultiRef™. The service provider identifies one station in the network as the master station. Raw observations of the master station, together with the between-station single-differenced corrections are transmitted to the rover.

Chapter 5 presents the distance weighted interpolation, plane interpolation, and collocation interpolation methods to interpolate the network corrections of each station onto the position of the rover receiver. The equation used to determine the parameters of these methods is given and the interpolation surfaces from same data set are shown.

In Chapter 6, the test results in the measurement and position domains, with and without the network corrections, are compared. Two typical scenarios are investigated, namely he ionospheric-free and geometric-free misclosures before and after applying network corrections. Also the network solutions using three interpolation techniques are compared with a single baseline solution in terms of accuracy and time-to-first-fix.

Chapter 7 presents the conclusions and some recommendations for further investigations. Some conclusions are made with respect to the bandwidth savings that is realizable via RTCM 3.0, the significance of data compression for network corrections, and improvements in achievement through network corrections in measurement and position domain. Some recommendations regarding ionospheric effects on network RTK positioning are given.

Chapter Two: RTCM 3.0 Overview

2.1 Motivation of RTCM standard

The U.S. Department of Defence (2001) states that the positioning accuracy of the Standard Positioning Service (SPS) is approximately 13 m in the horizontal and 22 m in the vertical at a 95% probability level (Liu 2003). However, the SPS cannot meet many civil applications requirements, such as numerous air and marine applications, which often requires sub-decimetre level accuracy at very high integrity, and surveying which requires sub-centimetre level accuracy (Lachapelle 2000). So, In order to achieve high-level accuracies, the Differential GPS (DGPS) techniques utilizing two GPS receivers' pseudorange observables and Double Difference (DD) techniques utilizing two GPS receivers' carrier phase observables must be used. The Radio Technical Commission for Maritime Services established Special Committee 104 (SC104) Working Group in the early 1980's to develop standards for differential services.

In addition, in order to enhance the application of GPS, a GPS modernization effort was announced by the U.S. government in January 1999. This initiative includes the addition of two new signals for civil use. One Coarse/Acquisition (C/A) signal located at 1227.60 MHz (L2C) and the other signal, located at 1176.45 MHz (L5), are now gradually becoming available (GPS Modernization 2005)

With GPS modernization, new relevant messages types should be added to the RTCM standard. Moreover, RTCM Version 2, which was developed more than ten years ago, is subject to increasing complaints for undesirable consequences of its parity scheme and data format. Therefore, a new standard is required that avoids RTCM Version 2's shortcomings; the forthcoming package of refinements must also readily accommodate Galileo, the new European GNSS system, new signals (i.e. L2C, L5) and new applications (i.e. network RTK positioning).

2.2 Problems of RTCM Version 2

RTCM Version 2 is patterned after the GPS data format, in terms of employing the same word size, word format and parity algorithm. The biggest difference is that RTCM Version 2 messages utilize a variable length message format depending on the number of satellites, whereas the GPS data has fixed length subframes. The reason why this parity algorithm and data format was chosen is quite straightforward, since this allows receivers to use the same parity checking algorithm as is used in the GPS navigation data message. However with the passage of time, the advantages of using the same scheme as GPS data diminish, since many GNSS and related systems, such as WAAS do not follow this scheme (Department of Transportation 2002). Moreover, compared to the many complaints related to inefficient data integrity, facility, and bandwidth usage associated with this parity scheme, the advantage of this scheme - reusing the GPS navigation data message parity checking module to save programming effort - seems insignificant (Kalafus & Van Dierendonck 2003).

RTCM Version 2 messages are composed of units of 30-bit words. Each message has two words of general message information as shown in Figure 2.1. After these two words of general message information follows variable length of messages composed of 30-bit words.

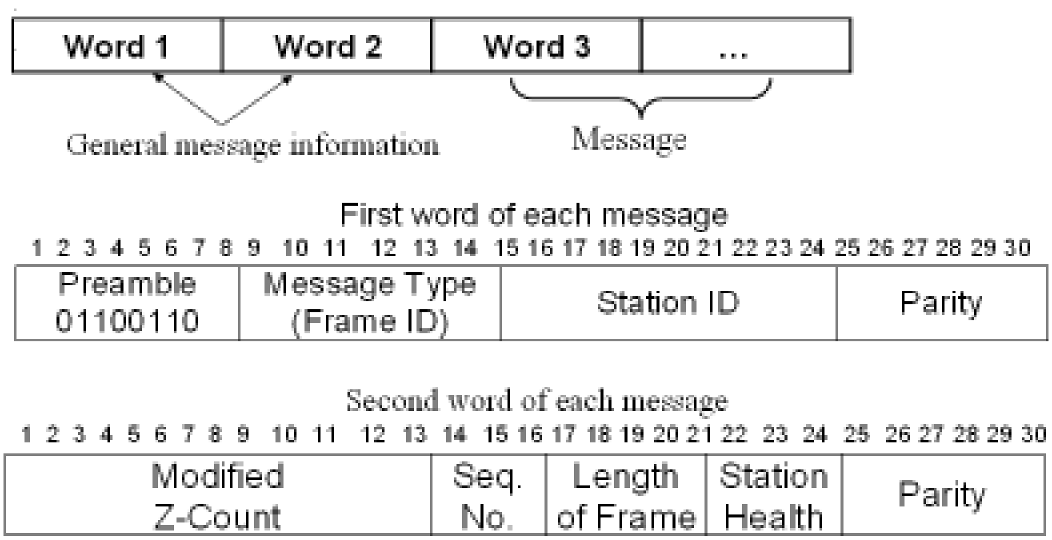

Figure 2.1: Data format of RTCM Version 2 (RTCM 2001).

Each 30-bit word contains 24 bits of data followed by 6 bits of parity, all of which are generated using an extended Hamming code (32, 26), so that each word's parity depends on the last two bits of the foregoing word (RTCM 2001). Despite the dedication of so many bits to parity (6 bits out of every 30-bit word), the actual integrity of the message is not very high because of the inherit characteristics of the Hamming code (32, 26). A comparison between the parity schemes of RTCM Version 2 and RTCM 3.0 is given in details in Section 2.4. Further disincentives to use this data scheme include the awkwardness of 30-bit words in product development scenarios, which are more in favour with 8-bit byte unit, as well as the limitations of the hamming code in that it is designed to handle noise-induced single-bit random errors and is therefore vulnerable to burst errors.

Furthermore, RTCM Version 2 results in the alteration of the meaning of message fields based on information contained in earlier fields. For example, message type 18, which contains the uncorrected carrier phase observations; the first two words for each message type are shown in Figure 2.1, while Figure 2.2 shows the remaining words for message type 18.

THIRD WORD

F	R	GPS TIME OF MEASUREMENT	PARITY

EACH SATELLITE - 2 WORDS

M	P/C	G	SID	DQ	CUM. LOSS OF CONT.	CARRIER PHASE UPPER BYTE	PARITY

CARRIER PHASE LOWER BYTES	

Figure 2.2: Message type 18 – uncorrected carrier phase message format (RTCM 2001).

The symbols are defined as follows:

F: Frequency Indicator (L1 or L2),

R: Reserved,

M: Multiple Messages Indicator,

P/C: C/A- Code/P-Code indicator,

G: GPS/GLONASS satellite constellation indicator,

SID: Satellite ID,

DQ: Data Quality,

Cum. Loss of Cont.: Cumulative Loss of Continuity Indicator.

The fields “Frequency Indicator” and “GPS/GLONASS satellite constellation indicator” will be combined together to indicate which carrier phase is inside the messages. Consequently, the field nominally identified as “Carrier Phase” could be represented by either L1 GPS, L2 GPS, L1 GLONASS or L2 GLONASS carrier phase measurements. However, RTCM 3.0 purports to define an entirely different message for each variation, i.e. one message type for GPS, one message type for GLONASS, and so on. Thus this specified format produces less ambiguous messages, although requiring a greater number of message types. Because RTCM 3.0 commences identification of messages at 1000, naming space is not a significant issue.

2.3 Components of RTCM 3.0

As described in the previous two sections, RTCM Version 2 is receiving a lot of complains about its awkward data format and inefficient parity scheme. RTCM SC104 is attempting to develop a new standard to overcome these problems and replace Version 2.

As of February 2004, Version 3 (referred to herein as RTCM 3.0) included only those message types that were validated in interoperability tests. These message types, i.e., message types 1001 to 1013, entitled GNSS RTK Messages (RTCM 2004a), will support traditional GNSS single station RTK and Differential GPS operations. These particular message types have successfully passed real-time interoperability tests performed at the Minnesota Department of Transportation Research Facility in Albertville, MN in 2003 (Kalafus & Van Dierendonck 2003)

However, a number of message types to support new applications, such as network RTK, have not been accommodated in the initial release of RTCM 3.0. Nevertheless, a document entitled *Version 3 Proposed Messages* (RTCM 2004b) lists these message types and categorizes them into the following eight groups. They are in different phases of development. Some like GNSS Network Corrections are under field tests, and some like GPS Ephemeris are still under internal discussion:

1. GNSS Network Corrections
2. GPS RTK Compact Corrections
3. GPS Differential Corrections
4. GLONASS Differential Corrections
5. Radiobeacon Almanac
6. ASCII Characters
7. GPS Ephemeris
8. Proprietary messages

GNSS Network Corrections are defined so as to facilitate implementation of GNSS RTK Networks, which are of great interest to researchers and industry. In such a process, the network service provider identifies one Master station in the network and transmits conventional information obtained at the master station through GNSS RTK messages. Additionally, the provider transmits network corrections for each satellite and each auxiliary station. Chapter Three contains a description of how the proposed RTCM 3.0 GNSS Network Corrections work with GNSS RTK messages to support network RTK.

The Compact RTK Correction messages further reduce the number of data bits of the messages by transmitting the difference between the computed geometric range and the adjusted phase range as already standardized in RTCM Version 2. The data transmission rate of these messages is very efficient, and thus could facilitate a limited broadcasting data link such as a radiobeacon to provide RTK services.

Differential GPS correction messages will transmit pseudorange corrections (PRC) in the same manner as defined in RTCM 2.3 and will support metre level accuracy DGPS services.

Although Differential GLONASS correction messages are similar to Differential GPS correction messages, as yet, they support only GLONASS Differential applications.

Radiobeacon almanac messages transmit the location, frequency, operational status, and station name for a network of marine radiobeacons. So, these messages types facilitate a GPS receiver to automatically transit between different differential data transmitters.

ASCII messages are defined based on Unicode, which is an international data standard that can be used to display messages in any language on a printer or screen.

GPS ephemeris information enables a reference station to broadcast ephemeris information to users in case the satellite ephemeris is inconsistent.

Proprietary messages, in the form of 95 message types from 4001 through 4096, are reserved for proprietary use by RTCM. RTCM will assign one message type to a company or organization for proprietary use. This grants the service provider the flexibility of combining general and proprietary messages on the same broadcast link to achieve superior performance. To date, proprietary message addresses have been assigned to selected companies, such as NovAtel Inc. (4093) and Leica Geosystems (4092), etc.

In addition, messages related to Galileo and the new signals (i.e. L2C, L5) are at a preliminary stage of development and are the subject of internal discussions within the RTCM Working Group.

2.4 New Message Format of RTCM 3.0

RTCM 3.0 defines a new frame structure that consists of a fixed preamble, a message length definition, the actual message, and a 24-bit Cyclic Redundancy Check (CRC), as shown in Table 2.1. The variable length data messages range from 0 to 1023 bytes. Generated from the preamble through to the end of Variable Length Data Field using a seed 0, the 24-bit QualComm CRC is located at the end of the message. A detailed description of this CRC algorithm is included in the RTCM 3.0 standard (RTCM 2004a). This CRC algorithm provides sufficient integrity for all but the most stringent safety-of-life applications (Kalafus & Van Dierendonck 2003). A comparison of the parity of RTCM Version 2 and Version 3 is shown in Table 2.2.

Table 2.1: Data format of RTCM 3.0 (RTCM 2004a)

Preamble	Reserved	Message Length	Variable Length Data Messages	CRC
8 bits	6 bits	10 bits	Variable length, integer number of bytes	24 bits
11010011	Not defined – set to 000000	Message length in bytes	0-1023 bytes	QualComm CRC-24Q

3 bytes (Preamble, Reserved, Message Length) 3 bytes (CRC)

Table 2.2: Parity of RTCM Version 2 and 3

Parity of RTCM Version 2	Parity of RTCM Version 3
Extended Hamming code (32,26) with distance of 4	Cyclic Redundancy Check 24 (also used in WAAS and to be used in L5)
Suited to situations in which random single bit errors are present	Protects against burst error as well as random errors
Detects all errors less than 4	Detects all single and double errors Detects any odd number of errors
An undetected error is produced from certain patterns of four errors	Detects any burst errors for which the length of the burst is $\leq$24 bits Detects most large error bursts with length >24 bits. The probability of undetected burst errors is $2^{-24} = 5.96 \times 10^{-8}$
Dependent on preceding word	Independent of other messages

2.5 GNSS RTK Messages

The composition of GNSS RTK message types is described in RTCM (2004a). It is highly flexible, highly efficient in terms of utilization of available broadcast bandwidth and in supporting conventional RTK operations. By utilizing this new standard, low-bandwidth broadcast data link could provide RTK services, which is impossible with the old standard RTCM Version 2.

2.5.1 Flexibility

GNSS RTK messages are structured into four groups: observation, station coordinates, antenna description, and auxiliary operation information, as described in Table 2.3. The various message types in each group contain similar information. The message types 1001, 1003, 1005, 1007, 1009, and 1011 contain the minimum information required to provide the service, while the message types 1002, 1004, 1006, 1008, 1010, and 1012 contain additional information for enhancing the performance of the differential service. Herein, the length of even numbered message types is larger than the odd numbered message types because they contain additional information. In the following, the odd numbered message types in the first three groups are called shorter messages, while the even ones are called longer messages. In practice, the service provider must transmit at least one message type from each of the first three groups, be they of the short or long varieties. Alternatively, the provider could combine use of shorter and longer messages to balance the performance and bandwidth. The last group contains only one message type; this message type summarizes all messages transmitted by the particular reference station, which is auxiliary to system operation.

The content of these messages are described in Table 2.4 to Table 2.6. Table 2.4 gives the message headers for message types 1001 to 1004, while Table 2.5 and Table 2.6 show the data field for each satellite in these message types. Message types 1001 and 1002 contain almost the same information, except that message type 1002 contains two additional fields. The first of these is “GPS Integer L1 Pseudorange Modulus Ambiguity”, which represents the integer number of full pseudorange modulus divisions (299,792.458 m) of the raw L1 pseudorange measurement. The second is “GPS L1 CNR”, which provides the carrier-to-noise ratio of the satellite’s signal in dB-Hz. Use of these two additional fields offers the potential to optimize accuracy and ambiguity resolution time, albeit at the cost of larger bandwidth. If throughput is not limited and the additional information is available, it is recommended to use the longer messages. Similarly, message type 1003 provides the minimum data set for L1/L2 operation, while message type 1004 provides the full data content to enhance performance of the service. In practice, however, the longer and shorter messages could be combined for use, because the additional information in the longer messages does not change very often in the context of a positioning mission. For example, the "GPS L1 CNR" in 1002 message type represents the Carrier-Noise Ratio of the signal strength, which should not vary with time frequently. The longer messages could be sent less often or whenever these additional information changes while all the other time, the shorter messages could be sent. In this way, comparable results could be achieved with a lower band rate.

Table 2.3: Four groups of RTCM 3.0 GNSS RTK messages (RTCM 2004a)

Group Name	Sub-Group Name	Message Type	Message Name
Observations	GPS L1	1001	L1-Only GPS RTK Observables
		1002	Extended L1-Only GPS RTK Observables
	GPS L1 / L2	1003	L1&L2 GPS RTK Observables
		1004	Extended L1&L2 GPS RTK Observables
	GLONASS L1	1005	L1-Only GLONASS RTK Observables
		1006	Extended L1-Only GLONASS RTK Observables
	GLONASS L1 / L2	1007	L1&L2 GLONASS RTK Observables
		1008	Extended L1&L2 GLONASS RTK Observables
Station Coordinates		1009	Stationary RTK Reference Station ARP
		1010	Stationary RTK Reference Station ARP with Antenna Height
Antenna Description		1011	Antenna Descriptor
		1012	Antenna Descriptor & Serial Number
Auxiliary Operation Information		1013	System Parameters

Table 2.4: GPS RTK message headers for message type 1001-1004 (RTCM 2004a)

Data Field
Message Number (e.g.,“1001”= 0011 1110 1001)
Reference Station ID
GPS Epoch Time (TOW)
Synchronous GNSS Flag
No. of GPS Satellite Signals Processed
GPS Divergence-free Smoothing Indicator
GPS Smoothing Interval

Table 2.5: Data fields for each satellite in message types 1001 and 1002 (RTCM 2004a)

	Data Field for Each Satellite	
	GPS Satellite ID	
	GPS L1 Code Indicator	
1001	**GPS L1 Pseudorange**	
	GPS L1 PhaseRange – L1 Pseudorange	**1002**
	GPS L1 Lock time Indicator	
	GPS Integer L1 Pseudorange Modulus Ambiguity	
	GPS L1 CNR	

Table 2.6: Data fields for each satellite in message types 1003 and 1004 (RTCM 2004a)

	Data Field for Each Satellite	
	GPS L1 PhaseRange – L1 Pseudorange	
	GPS L1 Lock time Indicator	
	GPS Integer L1 Pseudorange Modulus Ambiguity	**1004**
1003	**GPS L1 CNR**	
	GPS L2 CNR	

Through a proper selection of combinations of different message types, service providers can deliver a variety of different services, ranging from basic to more comprehensive positioning services (RTCM 2004a). The most basic service could be a GPS L1 only operation, in which only message type 1001 is broadcast along with basic station coordinates (1009) and antenna information (1011). This basic service cannot enhance the ambiguity resolution time because no CNR information is provided and the presence of a millisecond time ambiguity in the corrections. An example of a complete service could be a GPS/GLONASS dual system, dual frequency service including optimization of accuracy and ambiguity resolution time by providing CNR and unambiguous range observations. This kind of complete service could be achieved by a combination of messages types 1004, 1008, 1010, 1012 and 1013. Messages type 1013 is provided for rapid start-up and post-mission analysis (RTCM 2004a).

GNSS RTK messages avoid one of the RTCM Version 2's problems whereby the earlier flags altered the meaning of later fields, as shown in Section 2.2. The fields "Frequency Indicator" and "GPS/GLONASS satellite constellation indicator" are combined together in Version 3.0 to indicate which carrier phase is inside the messages. So the message types 18 could contain either L1 GPS, L2 GPS, L1 GLONASS or L2 GLONASS carrier phase measurements depending on these two indicators. However, RTCM 3.0 designates message type 1001: L1-Only GPS RTK Observables, 1003: L1&L2 GPS RTK Observables, 1005: L1-Only GLONASS RTK Observables, 1007: L1&L2 GLONASS RTK Observables. Thus, one message type 18 in RTCM Version 2 corresponds to four message types in RTCM 3.0, excluding the longer message types such as 1002, 1004 etc. This practice results in a clearer and unambiguous way to capsulate information at the cost of a larger number of message types. However, since RTCM 3.0 begins with 1000, it has up to 1000 message types available. Therefore, a large number of message types is not problematic.

2.5.2 Broadcast Bandwidth

Other than its operational flexibility, RTCM 3.0 institutes a wide range of efforts to compress the transmitted data. It avoids transmission of very large numbers to decrease the dynamic range of the data; for example, as in message type 1001 to 1004, the L1 pseudorange is divided into 1-millisecond lanes. For the basic service, only the factional part is transmitted while, for enhanced services, the 1-millisecond ambiguity is transmitted to optimize ambiguity resolution time. In this way, the range of pseudorange is reduced to 0 ~ 299,792.46 m which is much smaller than that defined in Version 2. Another data compression technique involves transmission of differences only, instead of absolute values. Thus, the L1 carrier phase is transmitted relative to the L1 pseudorange and L2 data is transmitted as differences with respect to L1 data. These data compression techniques aim at decreasing the number of information data bits and, hence, optimizing usage of the broadcast bandwidth.

As a further example of efficiency-oriented innovations, the 1003 message defined in RTCM 3.0 contains the same information (uncorrected carrier phase and pseudorange) as 18/19 in Version 2. However, using type 1003 saves a great deal of bandwidth as compared with the use of 18/19 when the same information is considered. The upper plot in Figure 2.3 shows how many bits are needed for transmission using these two standards versus the number of satellites. 1200 bits are enough for RTCM 3.0 to transmit ten satellites' information, whereas RTCM 2.3 does not allow transmission of dual frequency observations for three satellites within that limit.

Reductions in broadcast bandwidth and improvements in transmission effectiveness are secured not only by innovative data compression techniques, but also by the new data format of RTCM 3.0. The lower plot in Figure 2.3 shows the percentages of bits devoted to useful information pursuant to these two standards. Regardless of how many satellites are transmitted, only 80% of Version 2 data bits is useful, all the others serving only as parity bits. However in the Version 3.0 architecture, the percentage of useful information increases from 88% to 98% with the transmission of a greater number of satellites observations. Consequently, the percentage of parity bits decreases with an increasing number of satellites. Although fewer bits are devoted to parity, Version 3.0 offers much greater integrity control than Version 2 owing to its more advanced QualComm CRC-24 technique as shown in Section 2.4.

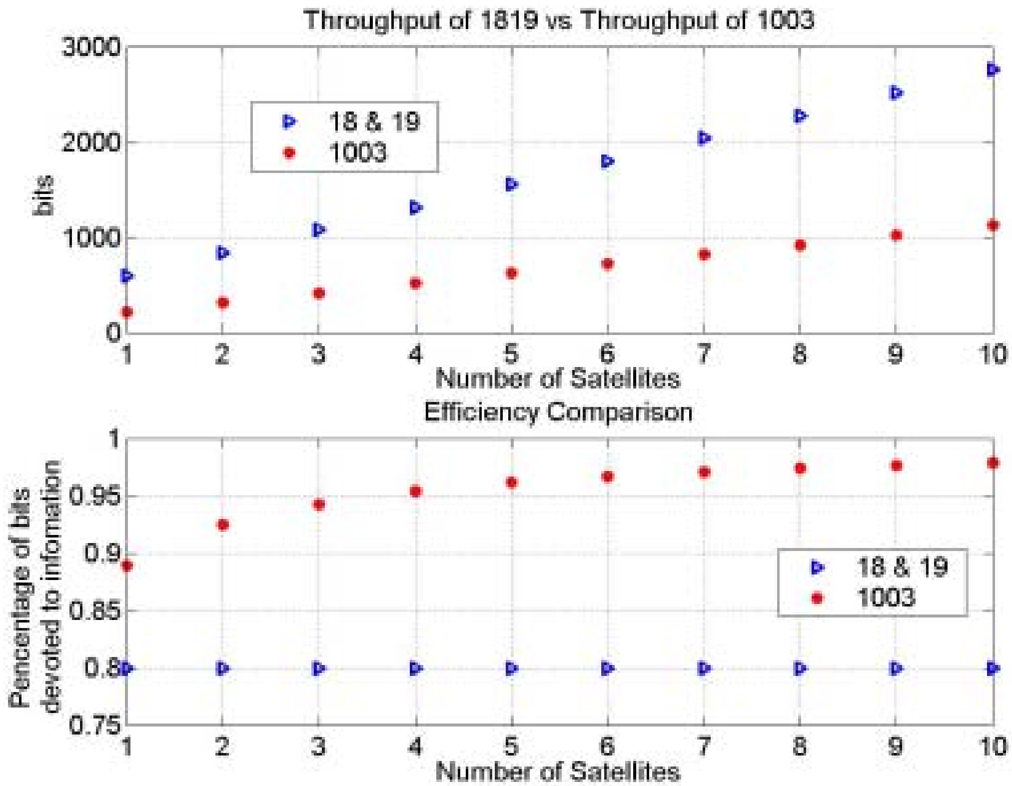

Figure 2.3: Bandwidth and efficiency comparison between RTCM 3.0 message type 1003 and RTCM 2.3 message types 18/19

It is plausible that RTCM 3.0 could eventually replace some messages in the RTCM 2.3 data framework. Some products, such as the Leica System 1200, can already apply RTCM 3.0 protocols for single station RTK positioning. Moreover, GNSS RTK messages also comprise a very important component for network RTK operation. A discussion of how GNSS RTK messages work together with network corrections to support a network RTK system is presented in Chapter Three.

Chapter Three: GNSS Network Corrections

As stated in Chapter Two, a number of message types that are not considered mature enough to be included into the RTCM 3.0 standard are classified as "Version 3 Proposed Messages" (RTCM 2004b). Among them, the GNSS Network Corrections Messages, which are designed to support network RTK applications, are intended for verification under final real-time interoperability tests. After confirmation via the real-time tests, these messages will be included as part of the RTCM 3.0 standard. Therefore, in this thesis, GNSS Network Corrections are always indicated as part of RTCM 3.0 for simplicity. As a fundamental part of RTCM 3.0 implementation in network RTK operation, "GNSS Network Corrections" will be the focus throughout the following sections.

In order to generate GNSS Network Corrections, some key concepts are introduced in the RTCM 3.0. These concepts help to reduce the sizes of the network RTK corrections, along with giving some hints for operation at the rover end on the application of these corrections. Chief among these concepts are: Common Ambiguity Level, Subnetwork, Master-Auxiliary, and Dispersive-non-dispersive.

3.1 Common Ambiguity Level

It is well established that the correct resolution of the double-differenced (DD) integer ambiguities is the key to high accuracy positioning for the single reference station approach. Similarly, the successful resolution of network ambiguities is the key to high accuracy positioning for positioning of the rover within a network. Assuming that the network software has resolved the ambiguities between each pair of permanent reference stations, the ambiguities can be removed from the original observations, and a common ambiguity level can be achieved across the network.

For example, for the case of two stations, only one correct double-differenced integer ambiguity is possible for one pair of satellites and one frequency. This number is always unique for one data set. However the Correction Difference (CD) is defined as a single difference value with respect to two stations. Therefore, a choice of the reference integer ambiguity is arbitrary but common for all satellites and therefore can be absorbed as a common clock error. The introduction of one cycle of reference integer ambiguity will raise the common ambiguity level by one cycle. Considering a multi-station network, if the ambiguities have been removed relative to one station deemed the master station, all stations within this network are on the same ambiguity level and thereby, all stations that have been on the same ambiguity level are considered to be in one *subnetwork*. For example, as Figure 3.1 shows, if both Ref2's and Ref3's ambiguities are removed relative to Ref1, these three stations are on the same ambiguity level and can be denoted by the same subnetwork ID. A more detailed description of relation between networks and subnetworks is given in Section 3.2.

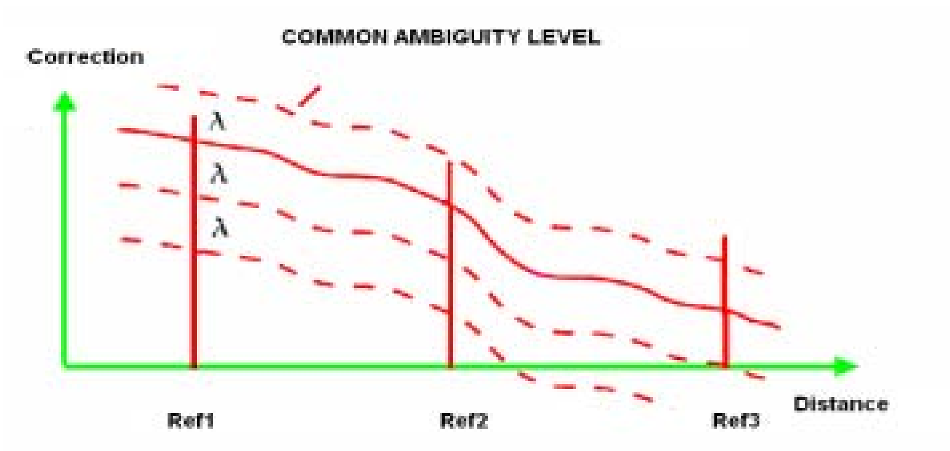

Figure 3.1: Common ambiguity level (λ is the wavelength of GPS signals)

Once the full set of information has been received, the rover receiver could determine how to apply this information. One option is to transfer back to the single difference approach instead of the network approach. As the rover possesses the full set of observations of the master station, and the correction differences between master and each auxiliary station, it can reconstruct the carrier phase observations of any station relative to the master reference station. Thus, a single difference approach could be utilized to compute the rover's position. Another approach would fully take advantage of network information. The rover receiver would spatially interpolate a part (or the complete suite) of the received corrections of the auxiliary stations to derive its own corrections relative to the master station. Then the rover would try to resolve the ambiguities between itself and the master station and compute its position.

3.2 Network and Subnetwork

The term subnetwork has been introduced in order to communicate to the user that not all reference stations in the network are currently on the same integer ambiguity level. In general, a network is defined as a group of stations that are likely to have a view of a same group of satellites and form a common ambiguity level. If a network is characterized by long inter-station distances, and it is not likely that the integer ambiguities bridging these distances can be resolved, the network should be separated into two or more networks instead of having several subnetworks all the time in one network (RTCM 2004c). For example, as shown in Figure 3.2, if the ambiguities between stations 14 and 2 are not likely to be resolved and in this case, it is better to separate the network into two smaller networks, each of which to be assigned a network ID to indicate two different networks, than to have two subnetworks all the time in one network. Having several subnetworks in one network is not necessarily beneficial. In other words, one network consists of only one subnetwork identified by one subnetwork ID in general (RTCM 2004c). When the common ambiguity level cannot be achieved in the network, the network can be divided into several subnetworks. Additionally, one subnetwork is normally associated with at least one master reference station, leaving the others as auxiliary reference stations.

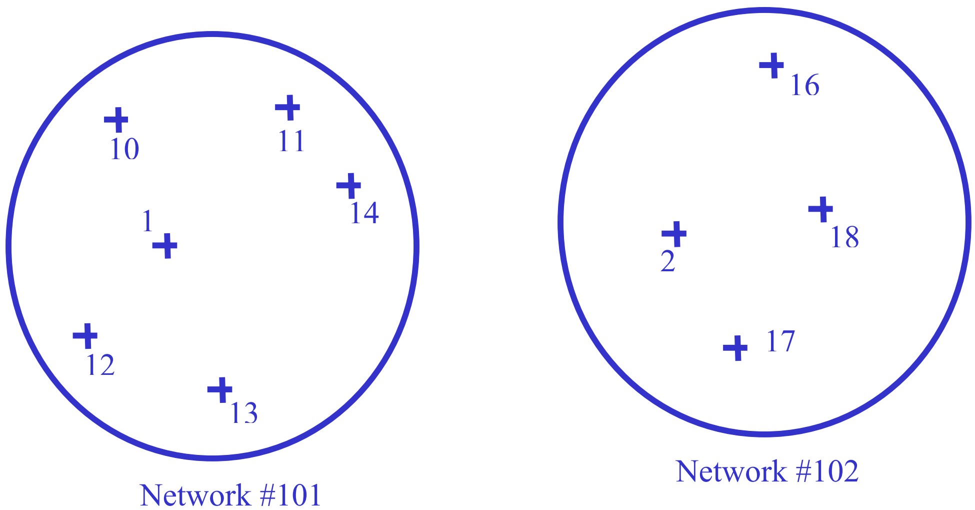

Figure 3.2: Separation of one network into two networks when necessary

However if station 15 is placed in the empty space between the two networks, as shown in Figure 3.3, it will bridge them. In this case, the entire collection of eleven stations becomes one network. This makes it possible that a rover in this area could use all the information from these 10 stations to contribute to its position solution.

As mentioned before, generally networks and subnetworks overlap each other as shown in Figure 3.3. When the network encounters difficulties to maintain a common ambiguity level among all the reference stations, i.e. one homogeneous integer ambiguity solution falls apart, the network will be separated into two subnetworks as shown in Figure 3.4. For example, if station 15 has communication problems with the control centre or its observations become unavailable for some reason, and there is still no possibility to resolve the ambiguities between station 14 and station 2, the 10 stations cannot be in the same subnetwork anymore. The network is divided into two subnetworks identified as

Subnetwork #1 and #2, which indicates the network is not on a homogenous solution and has two separated integer ambiguity levels.

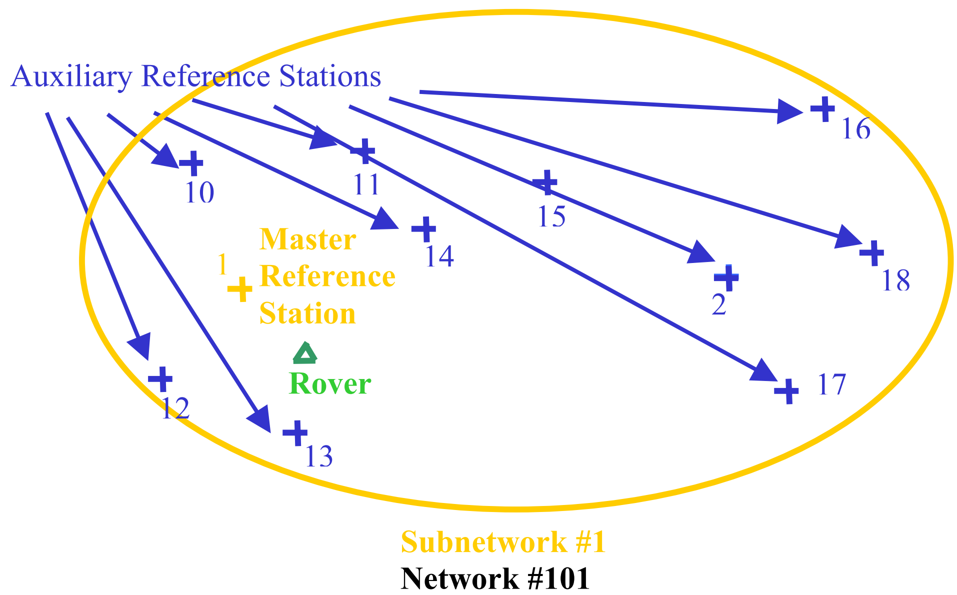

Figure 3.3: Network and subnetwork overlap each other

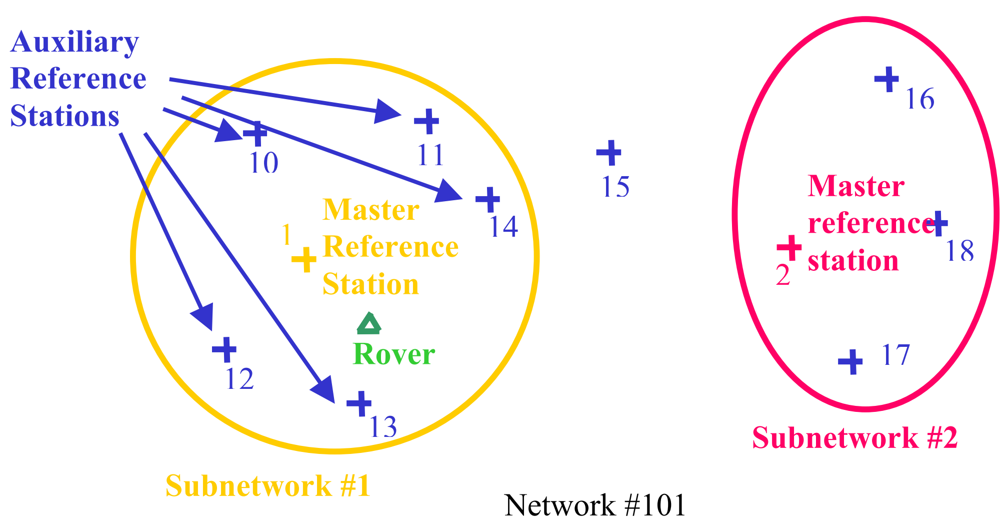

Figure 3.4: One network divided into two subnetworks

Eventually, when station 15 returns to the network, and the ambiguities between station 15 and station 1, as well as those between stations 15 and station 14, are resolved, all stations would be brought to a common ambiguity level again. Therefore the two subnetworks (Subnetwork #1 and Subnetwork #2 as in Figure 3.4) could combine again into one subnetwork (Subnetwork #1 as in Figure 3.3).

Generally, the service provider designates one station in one subnetwork to be the master station. Nevertheless, Figure 3.5 shows an example that one subnetwork has two master stations #1 and #2. In this case, not only do the blue stations act as auxiliary stations, but stations 1 and 2 act as auxiliary stations for one another. Each subnetwork has its own ambiguity level. They completely overlap each other.

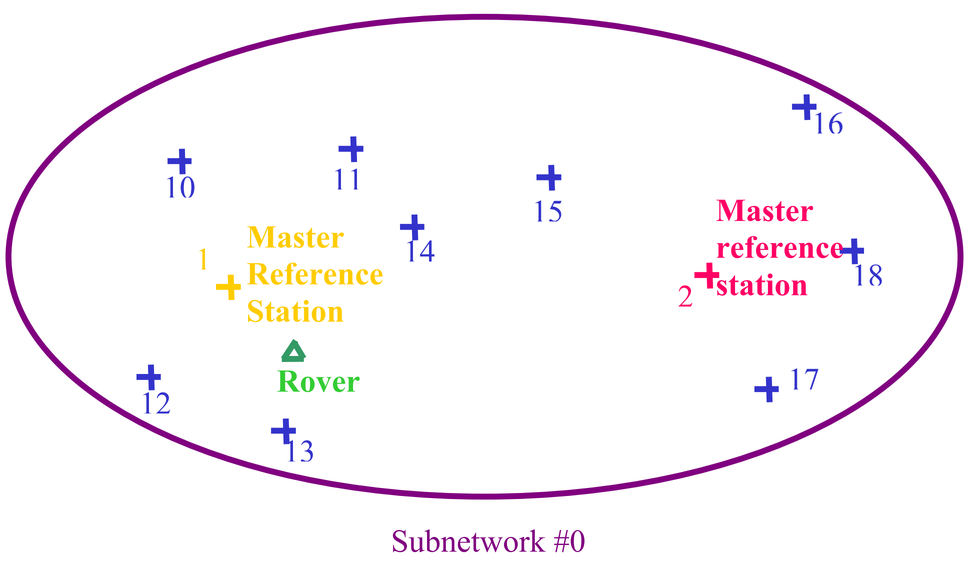

Figure 3.5: One subnetwork with two master reference stations

Multiple subnetworks may cause re-initialization at the rover end. For instance, the rover moves from subnetwork #1 into subnetwork #2, as shown in Figure 3.6. When the rover is located in the area of subnetwork #1, it is straightforward for it to use information associated with the Master Reference station and Auxiliary Reference stations inside subnetwork #1. As the rover exits subnetwork #1, it may use the same information source and in seeking an extrapolation of this data. Consequently, the resulting positioning accuracy may be degraded. If the rover continues to use the information from subnetwork #1 as it enters subnetwork #2, there will be no interruption; however, the positioning accuracy may be degraded further since the rover is so far away outside the valid area of subnetwork #1. The corrections from stations in subnetwork #1 will no longer model the errors that occur at the rover end. Therefore, the rover must to switch to subnetwork #2. However, since these two subnetworks are not on the same ambiguity level, the rover has to reinitialize its processing and reset its integer ambiguities. Thus discontinuity happens.

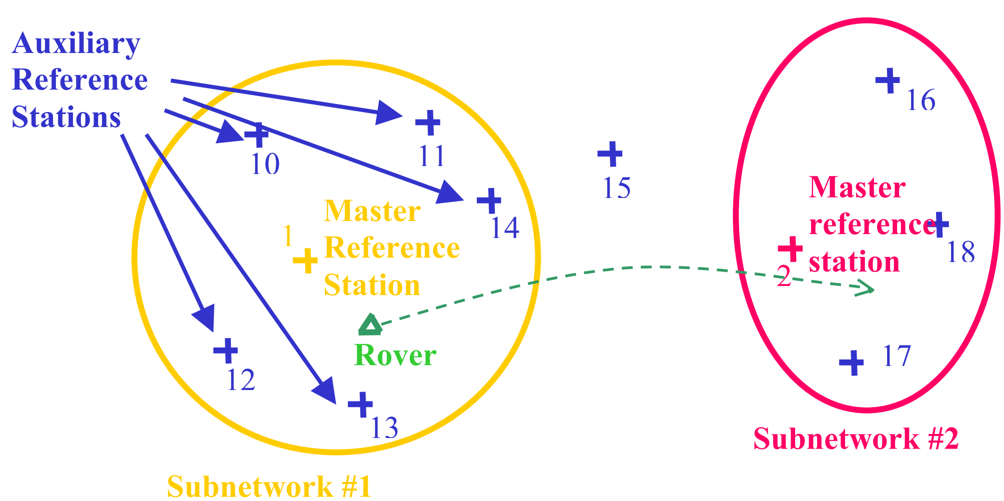

Figure 3.6: Rover across two subnetworks

In order to provide a continuous network service, service providers should not attempt to complete initialization of the network software on a frequent basis, optimally only about once a week or longer in practice. On the other hand, because generally it does not take a long time for network software to resolve ambiguities above 15 degrees across the network under a quiet ionospheric situation, it is recommended that service providers wait a couple of minutes until a homogeneous solution has been achieved in the network, i.e. until the network constructs one subnetwork. Success in resolving the ambiguities also depends on the scale of the network, that is the inter-station distances (RTCM 2004c).

3.3 Master-Auxiliary Concept

Normally, the network software will assign a master station in one subnetwork. All of the other stations will be deemed auxiliary stations. The ambiguities can then be removed relative to the master station, so that a common ambiguity level can be achieved. This is how the Master-Auxiliary concept comes into being. The concept uses the correction differences (i.e. between-station single difference corrections) to reduce the size of network corrections. These correction differences are the fundamental elements of GNSS network corrections in RTCM 3.0. In the following section, the basic undifferenced (raw) carrier phase equations are given first, followed by the between-station single difference equation to show the components of correction differences.

3.3.1 The Basic Carrier Phase Equation

The carrier phase observable, in units of cycles, can be written as

$$\phi = \frac{1}{\lambda}(\rho - \frac{I}{f^2} + T + m + c\delta t_{rec} - c\delta t_{sv} + \upsilon + \delta r_{sv} - \delta p_{rec}) + N \tag{3.1}$$

where

ρ : Range between satellite vehicle (at transmit time) and receiver (at receive time) (m)

I : Ionospheric delay parameter (= 40.30 TEC) (Hz^2m)

f : Carrier signal frequency (1575.42 MHz for L1, 1227.60 MHz for L2)

T : Measurement delay due to troposphere (m)

m : Measurement delay due to multipath (m)

c : Speed of light (m/s)

δt_{rec} : Receiver clock error (sec)

δt_{sv} : Satellite clock error (sec)

υ : Measurement noise (m)

λ : Wavelength of L1 or L2 carrier

δr_{sv} : Orbit error (m)

δp_{rec} : Receiver position error relative to the true positions (m)

N : Integer carrier-phase cycle ambiguity (cycles)

It is important to note that the positions of the reference receivers are assumed to be precisely known; that is, δp_{rec} is assumed to be zero. In addition, the reference receivers are installed at carefully selected places to ensure minimum multipath errors; moreover, these receivers are believed to have considerably small measurement noise, normally less than 0.1 percent of the wavelength. Consequently, the multipath and measurement noise will be neglected in the following deduction.

Equation 3.1 can be simplified and written as

$$\phi = \frac{1}{\lambda}(\rho - \frac{I}{f^2} + T + c\delta t_{rec} - c\delta t_{sv} + \delta r_{sv}) + N \qquad (3.2)$$

At the control centre, the network software will form double-differenced observables to estimate the double-differenced ambiguities between each pair of reference stations and satellites. These double-differenced ambiguities will be used to form the between-station single difference corrections.

3.3.2 Between-station Single Differences

For a pair of stations, namely the auxiliary station A and the master station M, a correction difference for one satellite, which is denoted by i, between these two stations can be generated as follows (RTCM 2004b), where Δ denotes the between-station single difference:

$$CD_{AM}^{i} = \Delta\rho_{AM}^{i} - \frac{c}{f}\Delta\phi_{AM}^{i} + c\Delta\delta t_{AM} + \frac{c}{f}\Delta N_{AM}^{i}$$
$$= \frac{\Delta I_{AM}^{i}(t)}{f^{2}} - (\Delta T_{AM}^{i}(t) + \Delta\delta r_{AM}^{i}(t)) \quad (3.3)$$

The correction differences are between-station single differences reduced by geometric range, receiver clock errors and ambiguities. The quantity remaining on the right-hand side of this equation is to be transmitted to the rover. It includes the single-differenced whole ionospheric errors, single-differenced whole tropspheric errors and orbital errors. This holds true for both L1 and L2 observations.

One problem arises, in that the single-differenced ambiguities cannot be determined due to the depleted model available at this point to remove the between-station single difference errors. Normally, only double-differenced ambiguities can be resolved. Note that

$$\nabla\Delta N_{AM}^{i,ref} = \Delta N_{AM}^{i} - \Delta N_{AM}^{ref} \Rightarrow \Delta N_{AM}^{i} = \Delta N_{AM}^{ref} + \nabla\Delta N_{AM}^{i,ref} \quad (3.4)$$

The single-differenced ambiguity can be determined only when ΔN_{AM}^{ref} is determined. ΔN_{AM}^{ref} is an arbitrarily selected integer, but common to all satellites. This would result in only a constant bias in all contributing correction differences of this pair of stations. Therefore, the constant bias will be cancelled out in any baseline estimation subsequently performed at the rover end or absorbed as a modified receiver clock error (RTCM 2004b).

3.4 Dispersive and Non-dispersive Components

The L1/L2 corrections difference between master station M and auxiliary station A, of one satellite i can be explicitly expressed as:

$$
\begin{aligned}
L1CD_{AM}^{i} &= \frac{\Delta I_{AM}^{i}(t)}{f_1^2} - (\Delta T_{AM}^{i}(t) + \Delta \delta r_{AM}^{i}(t)) \\
L2CD_{AM}^{i} &= \frac{\Delta I_{AM}^{i}(t)}{f_2^2} - (\Delta T_{AM}^{i}(t) + \Delta \delta r_{AM}^{i}(t))
\end{aligned}
\qquad (3.5)
$$

The ionosphere is a dispersive medium (Klobuchar 1996). From Equation 3.5, it can be seen that L1 and L2 will encounter different ionospheric errors, in a manner that is inversely proportional to the square of the carrier frequency; however, the tropospheric error and orbital error are the same for L1 and L2. Therefore, the L1/L2 correction difference can be resolved into two components: i.e., dispersive and non-dispersive parts, named respectively the Ionospheric Carrier Phase Correction Difference (ICPCD) and the Geometric Carrier Phase Correction Difference (GCPCD) by forming a geometry-free combination and an ionospheric-free combination (RTCM 2004b). In the following equations, the superscript and subscript for satellite i, auxiliary station A and master station M are omitted.

$$k_1 = \frac{f_2^2}{f_2^2 - f_1^2} = -1.545728, \quad k_2 = \frac{f_1^2}{f_1^2 - f_2^2} = 2.545728$$

$$\begin{aligned} ICPCD &= k_1 L1CD - k_1 L2CD \\ &= \frac{\Delta I(t)}{f_1^2} = \Delta I_1(t) \\ GCPCD &= k_2 L1CD + k_1 L2CD \\ &= \Delta T(t) + \Delta \delta r(t) \end{aligned} \tag{3.6}$$

One benefit of forming the ICPCD and GCPCD components comes from their different characteristics in terms of time. Wanninger (1999) has shown that the corrections in the mid-latitudes can reach 1.5 ppm per minute for the dispersive part and only 0.1 ppm per minute for the non-dispersive part. This means that the dispersive part (due to ionospheric effects) changes more rapidly than does the non-dispersive part (due to tropospheric and orbital effects). Moreover, a recent questionnaire completed by RTCM members suggested the following maximum tolerated latency values for the three effects: 120 seconds for orbital effects, 30 seconds for the troposphere, and 10 seconds for the ionosphere (Euler *et al* 2001). This difference supports the notion that the dispersive and non-dispersive components could be transmitted at different rates. Thereby, to break the L1 and L2 correction differences into dispersive and non-dispersive parts would yield a further reduction in the bandwidth, as transmission of the non-dispersive parts could be done at a lower rate than that of the dispersive part.

In summary, the network corrections defined in the RTCM 3.0 will transmit the ionospheric error (ICPCD), and the tropspheric and orbital errors (GCPCD) for each satellite at each auxiliary station relative to the master station on the same ambiguity level.

3.5 Broadcast Bandwidth

Additionally, as discussed by Euler *et al* (2001), the data range of these ICPCD and GCPCD is ±24 m, as compared to the data range of ±32,768 cycles for the phase correction defined by RTCM 2.3. Thus, RTCM 3.0 dramatically reduces the sizes of network RTK corrections compared to RTCM 2.3.

Figure 3.7 shows the throughput for transmitting ICPCD and GCPCD or, alternatively, transmitting the RTCM 2.3 20/21 messages versus the number of satellites and auxiliary stations in the network. Note that message 21 contains pseudorange corrections, which are not included in RTCM 3.0. However, messages 20 and 21 are usually, but not necessarily transmitted together. Thus, in simulation mode, both messages are transmitted. As shown in the Figure 3.7, an 80% bandwidth reduction can be obtained when 15 auxiliary stations and 10 satellites are considered.

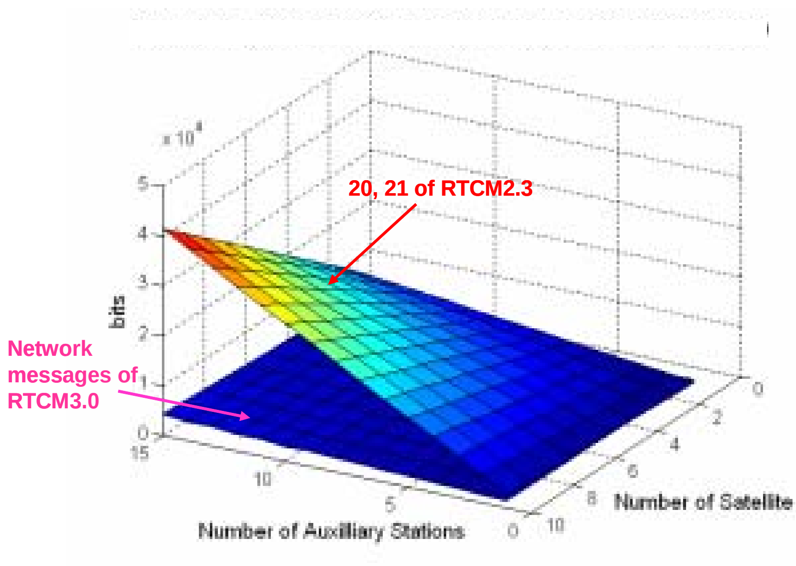

Figure 3.7: Throughput analysis of RTCM 3.0 GNSS network corrections and phase and pseudorange corrections of RTCM 2.3

Chapter Four: RTCM 3.0 in MultiRef™

This chapter describes how the RTCM 3.0 standard is integrated into a network software. Because of the different methods applied to the partitioning of responsibilities of network service software and rover RTK firmware, RTCM 3.0 implementation enables the network software to simply focus on resolution of ambiguities, while granting the rover the flexibility to utilize either a single reference station approach or network approach. The structures of network software using VRS and RTCM 3.0 respectively are discussed in detail. Also, an example of RTCM 3.0 data distribution in the context of the Southern Alberta Network Subset is presented.

4.1 Functions Partitioning of VRS and RTCM 3.0

As shown in Figure 4.1, there are five steps involved at the control centre and the rover. In a VRS implementation, as described in Section 1.2, the network software will first estimate the double-differenced network ambiguities and subsequently estimate the corrections at each station and each satellite; second, the corrections will be properly interpolated onto the position of the rover; third, the corrections will be transmitted to the rover receiver by generating a virtual reference station and the constructed observables of the virtual reference station will be transmitted to the rover via RTCM 2.3. Therefore, the network software located at the control centre completes the three steps above the magenta line in VRS implementation while, in RTCM 3.0 implementation, only Network Ambiguity Resolution needs to be accomplished at the control centre. This further enhances the flexibility of the rover's operation in the network since it possesses the full set of information pertaining to the network.

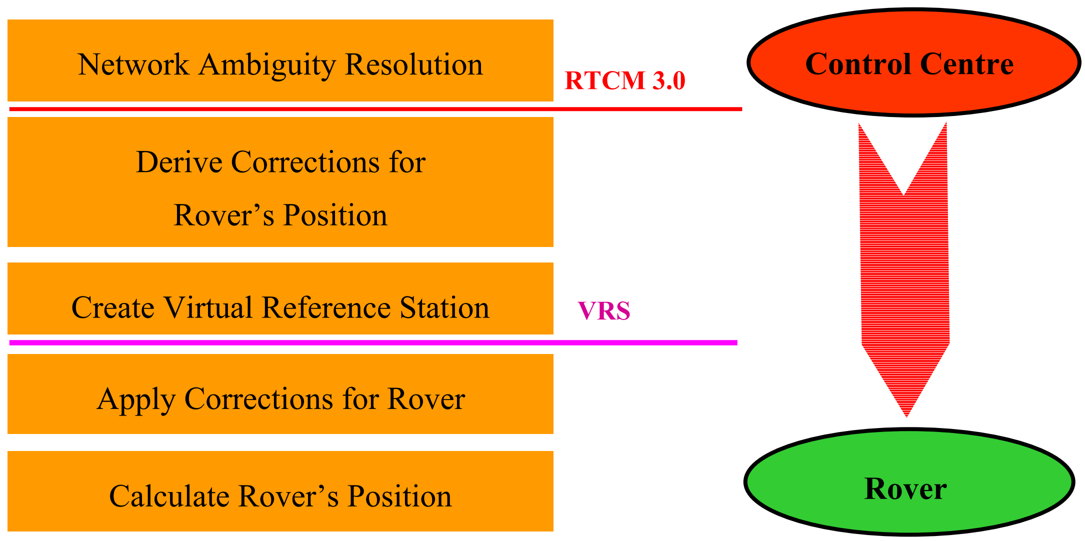

Figure 4.1: VRS and RTCM 3.0 function partitioning

4.2 Main Modules in MultiRef™

MultiRef™ is a Network RTK positioning software suite developed at the University of Calgary (Lachapelle et al 2000, Luo et al 2005). It is designed to produce the corrections for GPS reference stations and generate a VRS for each registered rover receiver in a GPS network to supply a regional network RTK service. The software utilizes state-of-the-art ambiguity resolution algorithms to resolve the network ambiguities and therefore generate network-based corrections. Based on the corrections at each physical reference stations and advanced least-squares collocation techniques, MultiRef™ will generate a virtual reference station for each rover. A description of the MultiRef^{TM} system's main modules, MRCor, MRNet, MRClient and MRUser, follows (Schleppe *et al* 2004).

4.2.1 MRNet

MRNet is a powerful tool for network data management. It collects the observables from the reference stations which are connected by internet through TCP/IP, UDP protocols. Meanwhile, it directly forwards the observables to MRCor. Also it provides an option to log the input data to file and archive the files on a daily basis for each input station for future post-mission processing if required. These files include the input data for the post-mission test described in Chapter Six.

4.2.2 MRCor

MRCor utilizes state-of-the-art ambiguity resolution algorithms to estimate the ambiguities throughout the network and produces satellite observation corrections for a set of given prediction points. It then forwards the grid of corrections at the prediction points to MRClient. However in this investigation, the MRCor is modified to generate RTCM 3.0 corrections at each station, except the designated master station. Details are given in Section 4.4.

4.2.3 MRClient and MRUser

MRClient will dynamically create MRUsers depending on how many registered rovers are present. Each MRUser uses the corrections from MRClient along with the approximate registered user receiver's position to create a virtual reference station. Then MRUser transmits the constructed VRS data to the rover receiver in RTCM V2.3 format using messages types 18/19 or 20/21.

4.3 MultiRef™ VRS Scheme

In view of the disadvantages of VRS implementation, the developers of MultiRef™ offer two operating options. One is in complete compliance with the traditional VRS scheme, while the other is a transformation of the traditional scheme.

4.3.1 Option 1: Users report to the control centre

In this configuration, all of the modules (MRNet, MRCor, MRClient and MRUser) are located at the control centre. Rovers in the network first register themselves to the control centre, then MRClient will create one MRUser for each rover. This configuration is shown in Figure 4.2.

In this option, the rover is completely blind to the fact that it is inside a network. In comparison to Option 2, this configuration does not need a secondary computer at the rover end. However, this configuration requires bi-directional communication between the control centre and the rover receivers. The one additional requirement imposed upon the rover is that it transmits its approximate position using TCP/IP, UDP, serial or radio to the control centre.

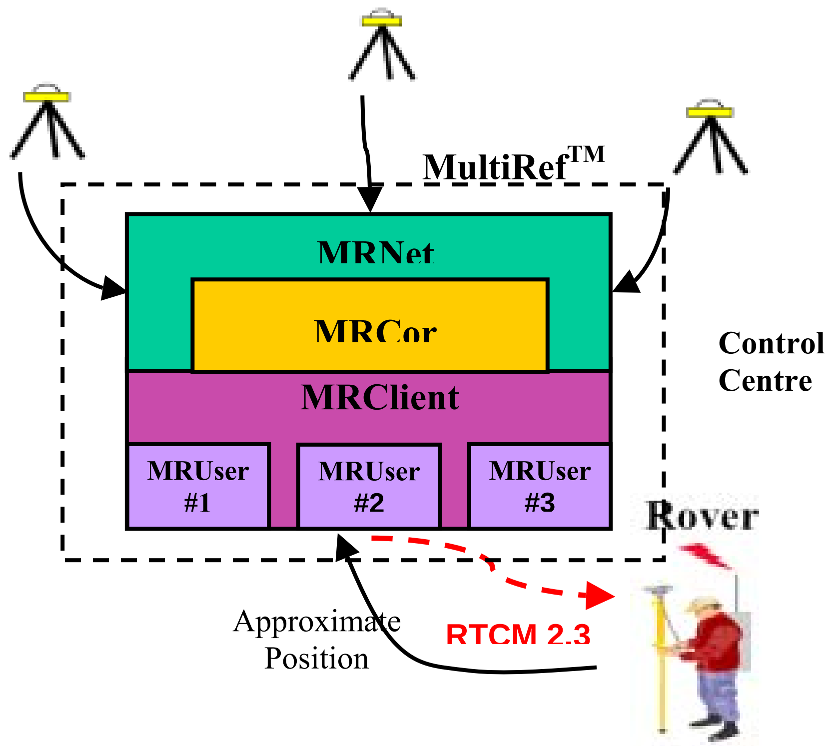

Figure 4.2: MultiRef^{TM} option 1 structure

4.3.2 Option 2: Control centre provides a correction grid

In this configuration, MRNet and MRCor are located at the control centre and MRClient/MRUser are located at the user end. MRCor transmits corrections on grid points in a self-defined format to the MRClient via TCP/IP, UDP or a serial link. Again each rover in the field transmits their approximate positions to MRUser (within MRClient). MRUser then outputs a virtual reference station to the rover receiver. However, in this configuration, as shown in the Figure 4.3, MRClient is installed on a secondary computer or laptop connected to the rover receiver, thus restricting the portability of rover receivers.

The main advantage of this configuration is that it is a broadcast system and there is no need for two-way communication between the control centre and the rover receivers. However, a secondary computer or laptop is required to run MRClient/MRUser.

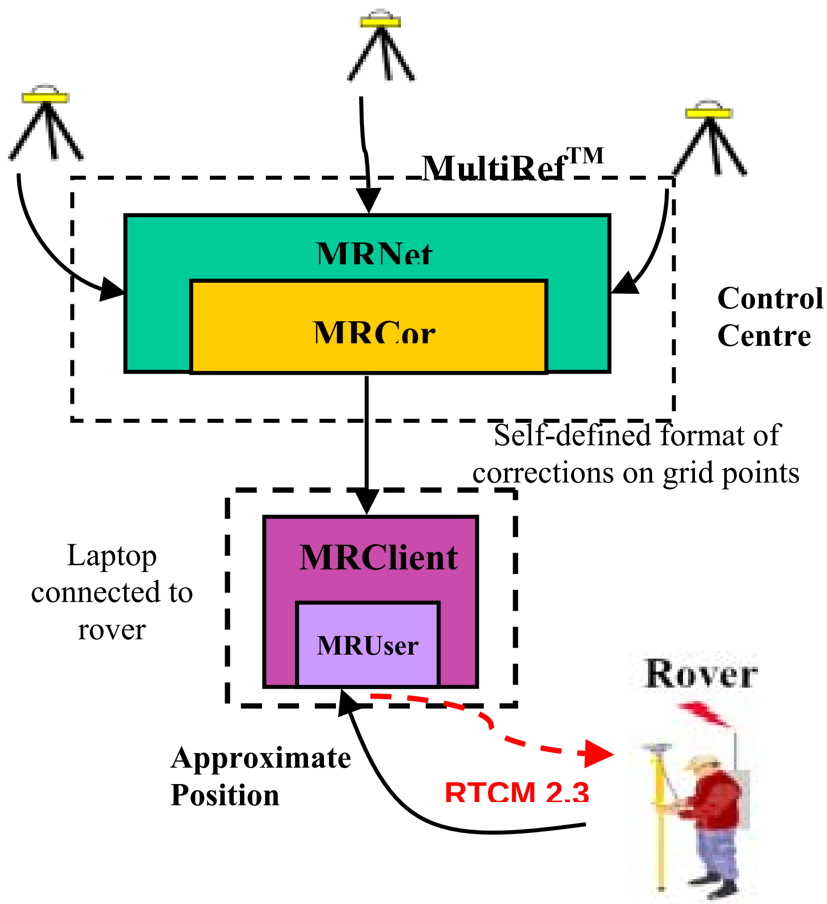

Figure 4.3: MultiRef™ option 2 structure

4.4 MultiRef™ RTCM 3.0 Scheme

The RTCM 3.0 scheme is, to some extent, similar to the Option 2 arrangement. The major differences between these two configurations are:

1) MRCor generates corrections on a grid of prediction points in Option 2, while, in the RTCM 3.0 scheme, MRCor outputs the corrections to the physical stations.

2) Distinct formatting conventions are used in Option 2 (self-defined format) between MRCor and MRClient. One of the responsibilities of MRClient is to interpret these self-defined messages. However, RTCM 3.0 scheme transmit standardized network corrections, the rover could interpret the network information and apply the data freely. In this way, neither MRClient nor MRUser are needed beyond this point. However, some of their functions will be implemented at the rover end, such as interpolation of the corrections.

3) The intrinsic limitation of the VRS still exists in Option 2, that is, the rover is blind to the fact that it is inside a network. Moreover, the rover is not informed of the quality of the VRS. By comparison, in the RTCM 3.0 scheme, as presented in Chapter One, the rover is aware of the network and can switch between network RTK and single baseline RTK. Also, by carefully flagging the network corrections, the rover has a good measure of the quality of network corrections.

As shown in Figure 4.4, in the control centre, MRCor provides the same function as before; it will resolve the ambiguities in the entire network and therefore generate corrections as appropriate, at each physical station. It then streams data to the RTCM 3.0 engine. The function of the RTCM 3.0 engine is to generate network corrections, convert data to RTCM 3.0 format and, finally, output data to the rover through the available data link.

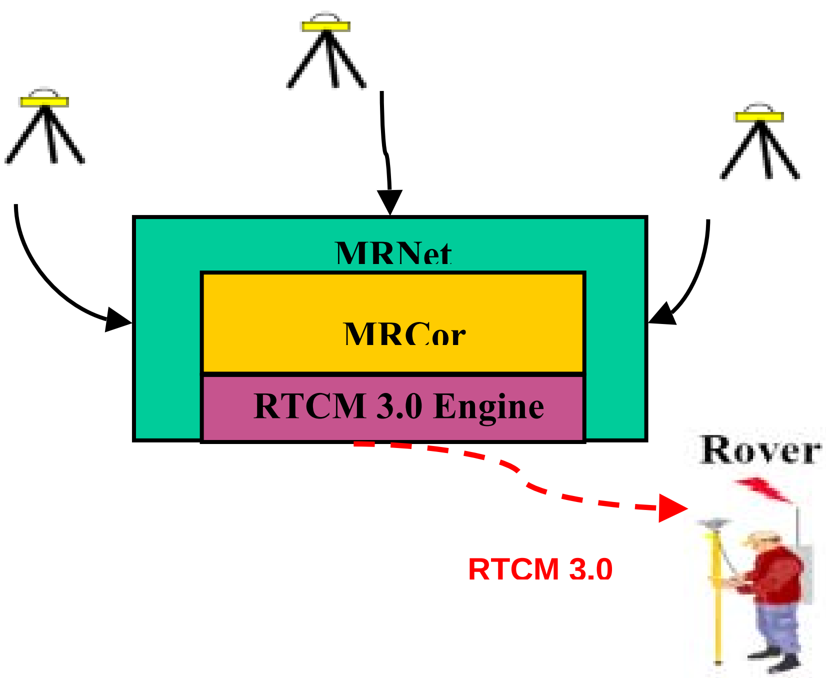

Figure 4.4: MultiRef™ RTCM 3.0 structure

Basically, two groups of RTCM 3.0 messages will be transmitted.

1) GNSS RTK MESSAGES, which transmit the raw observations and absolute coordinates of the master station. An appropriate combination of message types 1001 to 1013 is selected. In this case messages types 1004, 1006 and 1008 were selected.

2) GNSS NETWORK CORRECTIONS, which transmit the ICPCD/GCPCD of each auxiliary station (messages types 1102 and 1103), and coordinate differences between them and the master station (messages types 1101).

4.5 Southern Alberta Network

The Southern Alberta Network (SAN) is a GPS network established and maintained by the PLAN Group. It consists of 14 NovAtel Modulated Precision Clock (MPC) receivers located in Southern Alberta. The dimensions of the network are approximately 150 km north-south and 200 km east-west. In this test, only a subset of the SAN network (five stations: IRRI (0), COCH (1), STRA (2), BLDM (3) and AIRD (4)) serves as reference stations, with one receiver (at UOFC) designated as a rover and IRRI as the master station. The configuration of this subset of the SAN is shown in Figure 4.5. The baselines used for the network ambiguity resolution approach are the shortest set of independent baselines. They range from 28 to 59 km, as listed in Table 4.1

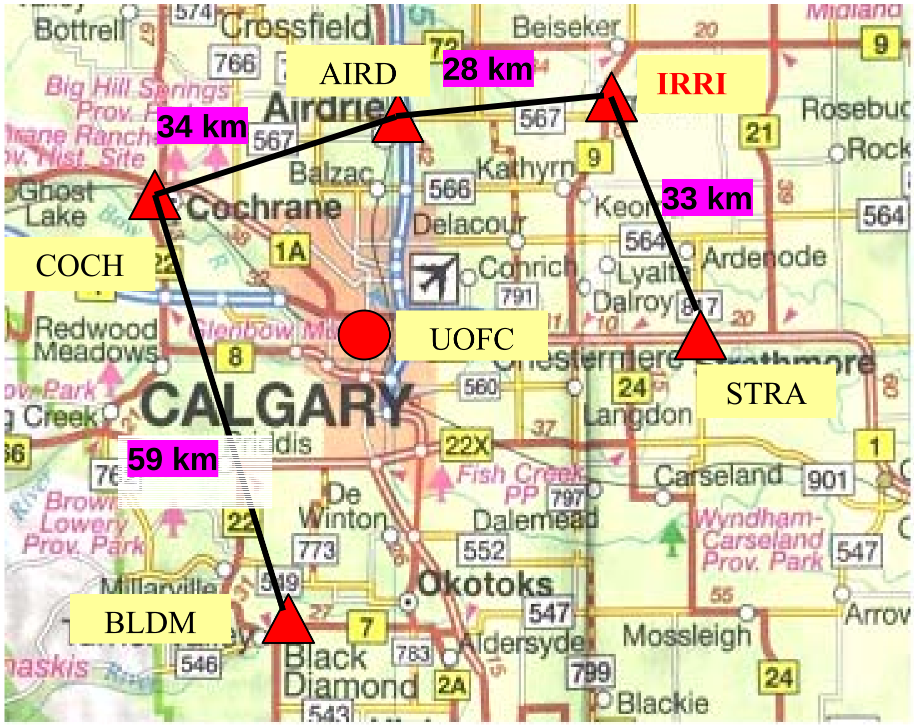

Figure 4.5: Subset of Southern Alberta Network (SAN) (Dao 2005)

Table 4.1: Baseline distances in the subset of SAN

Baseline	IRRI- STRA	IRRI- AIRD	AIRD- COCH	COCH- BLDM
Distance (km)	33	28	34	59

4.6 Message schedule

Since this is a small to medium scale network with only five stations involved, only one subnetwork that overlaps the network is considered throughout the tests.

In this particular network/subnetwork configuration, six types of RTCM 3.0 messages are scheduled to be transmitted: 1004 – raw observations of the master station; 1006 - Coordinates of the master station; 1008 - Antenna information; 1101 - Coordinates of the auxiliary stations relative to the master stations; 1102 - ICPCD of the auxiliary stations; and 1103 - GCPCD of the auxiliary stations. A transmission schedule detailing the sequencing of these messages is shown in Table 4.2.

The raw observations made by the master station IRRI (0) will be transmitted every second, while the coordinates and antenna information of the master and auxiliary stations will be transmitted every 10 seconds at different epochs. The ICPCD of all auxiliary stations will be updated every second; the GCPCD of stations COCH (1) and STRA (2) will be transmitted on odd seconds; the GCPCD of stations BLDM (3) and AIRD (4) will be transmitted on even seconds to equitably distribute the data within the operation period. Therefore, for this five-station network, the maximum bandwidth of this schedule would be 3752 bps if ten satellites were considered. How quickly these messages are updated depends on the bandwidth limitation. If a larger network is used, the messages will be updated less often.

One day's worth of data, observed on May 24, 2004, was selected to show the network corrections, ICPCD and GCPCD, as defined in Section 3.4. Figure 4.6 shows the ICPCD and GCPCD of auxiliary station BLDM. Figure 4.7 shows the double-differenced corrections between the auxiliary station and the master station, IRRI, which should equal the double-differenced misclosures (Gao 2004) of these two stations. The double-differenced misclosures defined herein are the difference between the double-differenced measurements minus the geometric ranges and the double-differenced ambiguities. The upper plot shows the DD ionospheric misclosures, that is, the dispersive component thereof; the lower plot shows the orbital and tropospheric misclosures, being the non-dispersive component. The baseline between BLDM and IRRI is about 70 km long; the ionospheric misclosures range from 0.5 to 2 cycles, which indicates that the differential ionospheric errors range from 1 to 5 ppm. It must be noted that the DD non-dispersive misclosures shown here are not reduced by any tropospheric model. Hence, the magnitude of the DD non-dispersive corrections is relatively large. If these corrections were reduced by a recognized tropospheric model, the magnitude should decrease substantially. However, as stated in Section 1.2, the RTCM formats require that any empirical atmospheric correction models should not be applied to the reference station data in order to eliminate the possibility of inconsistencies between different employed models.

Table 4.2: Transmission schedule example of RTCM 3.0 messages in a five-station network

second	GPS RTK Messages (Mater Station)			Network RTK Messages (Auxiliary Stations)		
	1004 Obs	1006 Coord.	1008 Antenna	1101 Coord.	1102 ICPCD	1103 GCPCD
1	(0)	(0)			(1)(2) (3)(4)	(1)(2)
2	(0)		(0)		(1)(2) (3)(4)	(3)(4)
3	(0)			(1)	(1)(2) (3)(4)	(1)(2)
4	(0)			(2)	(1)(2) (3)(4)	(3)(4)
5	(0)			(3)	(1)(2) (3)(4)	(1)(2)
6	(0)			(4)	(1)(2) (3)(4)	(3)(4)
7	(0)				(1)(2) (3)(4)	(1)(2)
8	(0)				(1)(2) (3)(4)	(3)(4)
9	(0)				(1)(2) (3)(4)	(1)(2)
10	(0)				(1)(2) (3)(4)	(3)(4)
11	(0)	(0)			(1)(2) (3)(4)	(1)(2)
12	(0)		(0)		(1)(2) (3)(4)	(3)(4)
13	(0)			(1)	(1)(2) (3)(4)	(1)(2)

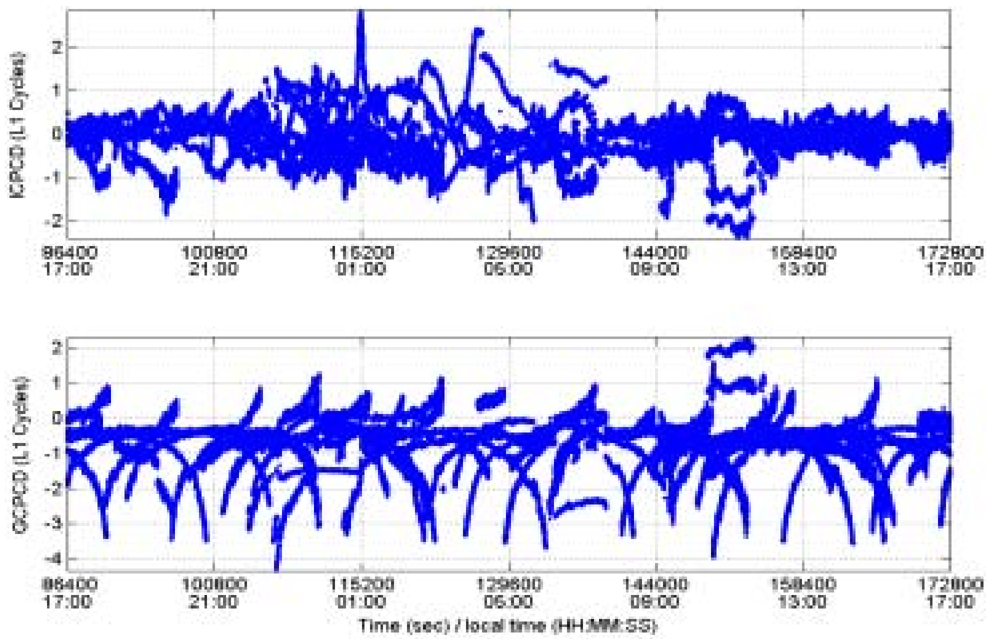

Figure 4.6: ICPCD and GCPCD of auxiliary station BLDM

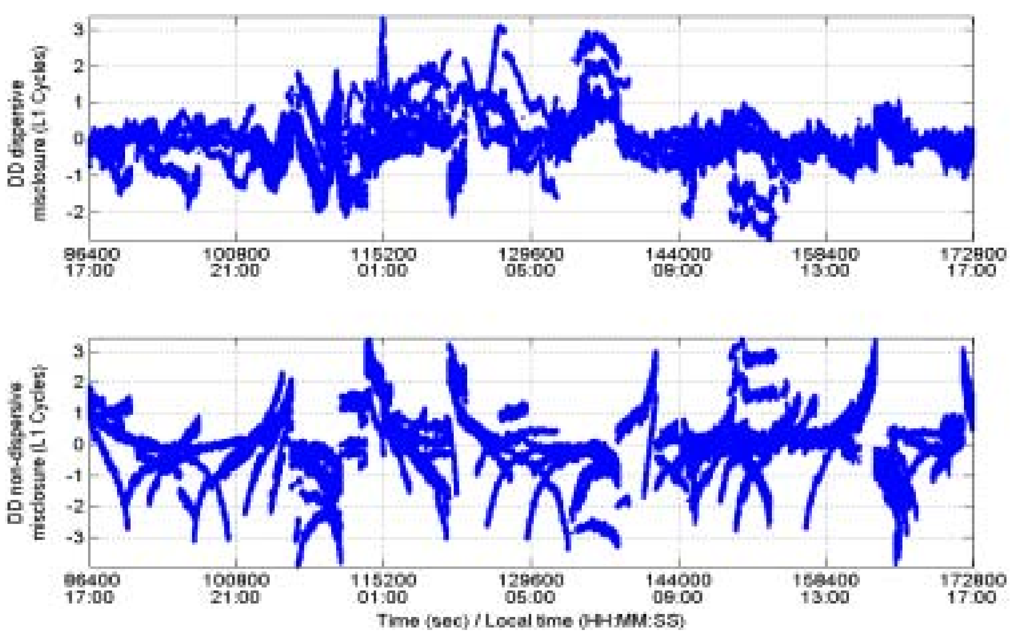

Figure 4.7: Double-differenced dispersive and non-dispersive misclosures with full tropospheric error

Chapter Five: RTCM 3.0 Implementation At Rover

Unlike other network RTK implementation schemes such as VRS and FKP, which employ RTCM Version 2, the RTCM 3.0 scheme transmits well defined network corrections to rovers, tasking them with interpreting and applying the appropriate corrections. The rover must decode the RTCM 3.0 messages and interpolate network corrections to its own position in order to deduce its own local errors. Thus, coordination of the corresponding components becomes important within the baseline processing strategy. In this chapter, three interpolation methods are given, namely distance-weighted, plane and collocation interpolation.

5.1 Applying Network Corrections at Rover End

A post-mission test was set up as shown in Figure 5.1. The network software MutiRef™ receives observations from each active reference station, estimates the ambiguities and generates the RTCM 3.0 messages as described in Chapter Four. Then a decoder and an interpolation function, both developed by the University of Calgary's PLAN Group, interprets the messages and interpolates the CDs to the position of the rover, which will reduce the double-differenced phase errors between the master stations and the rover. Note that, in RTCM 3.0, the GCPCD contains the full tropospheric error. Removing the tropospheric model error from the GCPCD before interpolation is recommended; i.e., the 'interpolate' function will deal only with the full ionospheric error and tropospheric residuals. The stepwise process developed in this research is as follows:

1) Decode the ICPCD and GCPCD from the RTCM 3.0 messages.
2) Reduce the GCPCD of each auxiliary station using the Hopfield tropospheric model based on a set of default parameters: pressure as 1023 Pa, relative humidity as 0.5 and Temperature 293° K using the available ephemeris.
3) Interpolate the ICPCD and reduced GCPCD to the rover's location to generate the corrections at the user position.
4) Compute the single differenced tropspheric error between rover and master station using the Hopfield tropspheric model and apply this component to the interpolated GCPCD at the user position.
5) Linearly combine ICPCD and full GCPCD to obtain L1CD and L2CD.
6) Read the observations of the master station and apply the L1CD and L2CD to L1 and L2 carrier phase, respectively.
7) Generate the corrected master station observation RINEX file.
8) The corrected master station observation RINEX file. together with the rover observation file are fed into GrafNav™ 7.01, a baseline processing software developed by Waypoint Consulting Inc., to perform RTK positioning.

In practice, the step 4 could be done either within this process or later, immediately before double differencing of the carrier phase. However, in this test, GrafNav™ 7.01, does not permit selection of the tropospheric model. If step 4 is not performed in the interpolation module and GrafNav™ is left to reduce the tropospheric error using its own tropospheric model, discrepancies may occur based on differences between the respective tropspheric models used in GrafNav™ and interpolation module. Therefore, to eliminate the risk of different tropspheric models being used at the interpolation module and subsequent baseline processing, step 4 is done at the interpolation module and GrafNav™ is set to disable the tropspheric model when processing the corrected master station RINEX file.

Because GrafNav™ can receive only general reference station observables as input, the CDs at the rover position will be applied back onto the master station observables and output in RINEX format (steps 6 & 7) to feed into GrafNav™ for dual frequency processing (step 8). It should be noted that, although the rover used in the following tests is actually static, it is set to kinematic mode in GrafNav™. Therefore, GrafNav™ will treat the rover as a kinematic one. This is very convenient to assess "kinematic" performance because the actual reference coordinates of the rover do not change and are known.

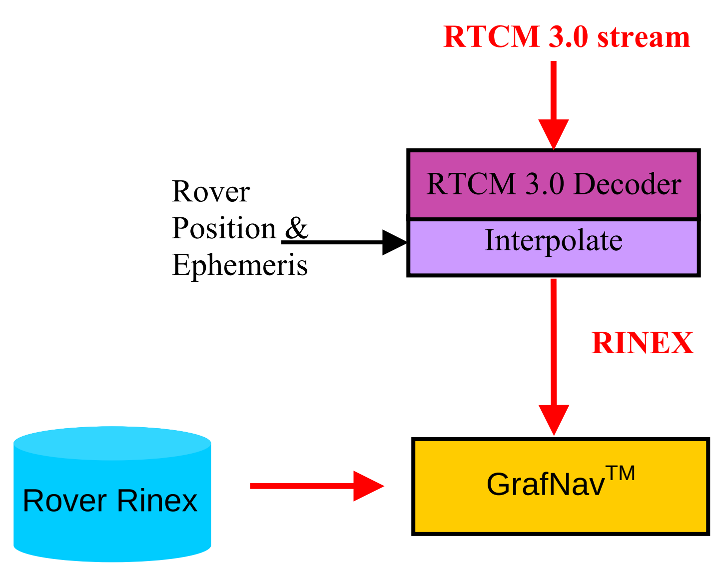

Figure 5.1: Post-mission test scheme at rover

Even if the corrections in the RTCM 3.0 stream (i.e. the ICPCD and GCPCD of each physical station) are precise, the spatial interpolation stage may introduce additional errors. The nature and severity of this class of errors depends on how well the interpolation methods match the combination of actual ionospheric, tropospheric and orbital error characteristics over the network area during the period of operation.

Three candidate interpolation techniques are discussed in the following section. These are the distance-weighted, plane and collocation methods.

5.2 Distance Weighted Interpolation

The distance-weighted interpolation method described in Euler & Zebhauser (2003) can be used whenever more than one station is available. The correction differences for the rover can be interpolated by following equations:

$$CD_{rover} = \frac{\sum_{i=2}^{n} CD_i / S_i}{\sum_{i=1}^{n} S_i} \tag{5.1}$$

where:

1 denotes the master station;

2 to n denote auxiliary stations; and

S_i denotes the distance between the rover and auxiliary station i.

Figure 5.2 shows the interpolation surface using the distance-weighted method that is generated from the four points shown in the figure. Among these points, [0,0,0] is referred to as the master point. The first two digits denote the horizontal coordinates of these points, while the last digit represents the CD at that point. The altitudes of these points are assumed to be the same.

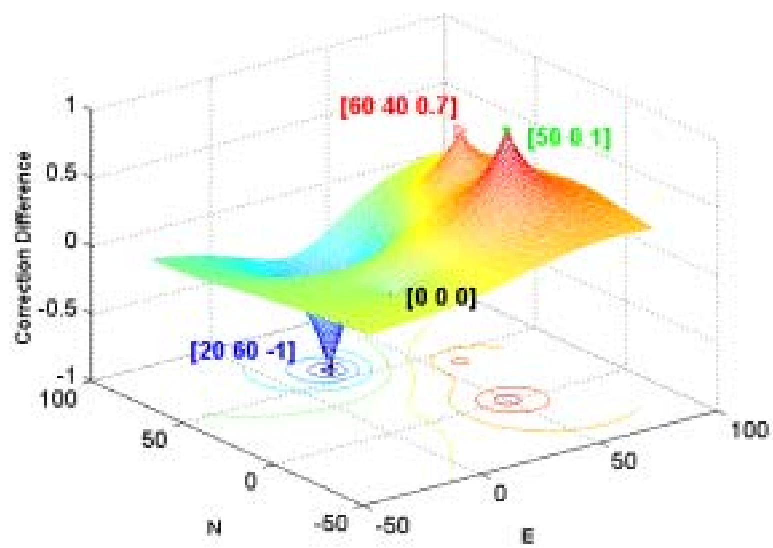

Figure 5.2: Distance weighted interpolation surface

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5.3 Plane Interpolation

One of the simplest linear interpolation methods consists of fitting a plane to the available data. In this method, the CDs at the rover are assumed to be lying on a plane surface, which is determined by the CDs of the auxiliary stations. The coefficients of a surface determined by three CDs would be calculated as follows:

$$\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = (A^T A)^{-1} A^T \begin{bmatrix} CD_1 \\ CD_2 \\ CD_3 \end{bmatrix} \quad A = \begin{bmatrix} E_1 - E_0 & N_1 - N_0 \\ E_2 - E_0 & N_2 - N_0 \\ E_3 - E_0 & N_3 - N_0 \end{bmatrix} \tag{5.2}$$

The correction difference at the desired position using the coefficients calculated above can be calculated as follows (Euler *et al* 2002):

$$\begin{aligned} &CD(E,N) = a_0 \times (E - E_M) + a_1 \times (N - N_M) \\ &a_0 \quad a_1 \qquad \textit{coefficients defining the surface} \\ &E_M, N_M \quad \textit{easting and nothing of master station} \end{aligned} \tag{5.3}$$

The problem associated with this method is that, if more than two auxiliary stations are used in the interpolation, discrepancies occur at station points. Figure 5.3 shows the interpolation surface determined by the same four points used in the plane interpolation method. In this figure, the red point [60 40 0.7] is above the surface, while the green [50 0 1] and blue [20 60 -1] points are under the surface. Only the master point are lying on the interpolation surface.

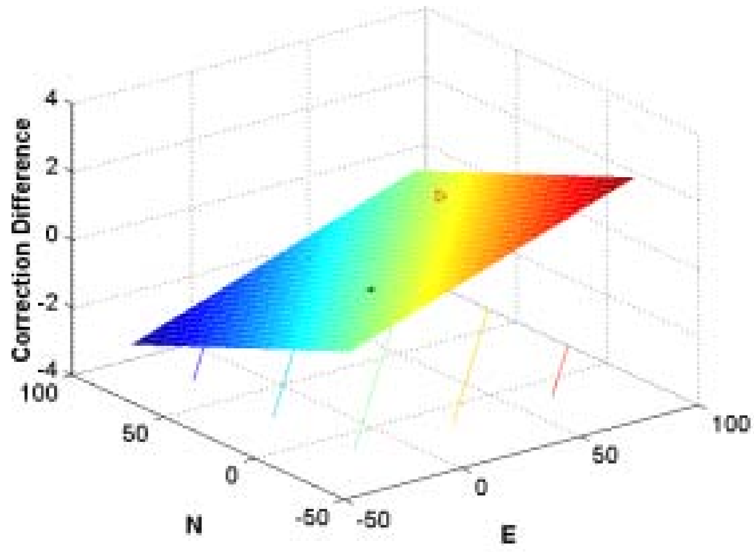

Figure 5.3: Plane interpolation surface

5.4 Collocation Equations

Least-squares collocation, also called least-squares prediction, uses a set of measurements (which are assumed to include deterministic signal and random noise components) from observation points, to calculate the "signal" at the prediction point (Raquet 1998). In this context, the signal used is actually the set of correction differences, with the prediction point at the rover's approximate position.

5.4.1 Least-Squares Collocation Basics

Least-squares estimation is a standard method used to estimate a unique set of values for a set of unknown parameters (x) from a redundant set of observables (l) based on a known mathematical model (f).

$$f(x,l)=0 \tag{5.4}$$

A conditional adjustment is a subset of this function, in which there are no unknown parameters (x), as represented by (Raquet 1998)

$$f(l)=0 \tag{5.5}$$

The least-squares condition adjustment generates measurement corrections $\hat{r}^*$ which are applied to l^* the measurements both at prediction points and measurement points using

$$\hat{l}^* = l^* + \hat{r}^* \tag{5.6}$$

This corrected measurement $\hat{l}^*$ meet the conditions given in Equation (5.5), i.e.,

$$f(\hat{l}^*)=0 \tag{5.7}$$

In a least-squares collocation problem, the estimate corrections $\hat{r}^*$ should be determined under the condition that

$$\hat{r}^{*T} C^{*-1} \hat{r}^* = \text{minimum} \tag{5.8}$$

where the hyper-covariance matrix can be expressed as (Krakiwsky 1990):

$$C^{*-1} = \begin{pmatrix} C_{SS} & C_{Sl} \\ C_{lS} & C_{ll} \end{pmatrix}^{-1} \tag{5.9}$$

where S denotes the signal at the prediction point, C_{SS} is the covariance matrix of the signals, C_{ll} represents the variance-covariance matrix of the observables; and C_{lS}, C_{Sl} are the covariance matrices between signals and observables.

The final estimates can be written as:

$$\begin{aligned} \hat{r}^* &= -C^* B^T [BC^* B^T]^{-1} w, \ where \quad w \equiv f(l) \\ B &\equiv [0_p \quad \& \quad \frac{\partial f(l_n)}{l_n}] \end{aligned} \tag{5.10}$$

Given the least-squares collocation basics above, the following sections applies the method to the specific case at hand to interpolate the network CDs at physical stations to the rover's position.

5.4.2 Solution to Network Collocation Prediction

The phase measurement can be expressed as follows, as per Section 3.3.1:

$$\phi = \frac{1}{\lambda}(\rho - \frac{I}{f^2} + T + m + c\delta t_{rec} - c\delta t_{sv} + \upsilon + \delta p_{sv} - \delta p_{rec}) + N \tag{5.11}$$

Note that δp_{sv} is used instead of δr_{sv} to represent the orbital error. If the calculated range is subtracted from this measurement, a nominal value of T is removed using a tropospheric error model. With δp_{rec} set to zero since the reference station's coordinates are known accurately, the measurement-minus-range observable can be represented as:

$$\bar{\phi} = \frac{1}{\lambda}(T^{'} - \frac{I}{f^2} + c\delta t_{rec} - c\delta t_{sv} + m + \upsilon - \delta p_{sv}) + N \tag{5.12}$$

where $T^{'}$ indicates the residual tropospheric error, after subtracting the modeled error. Thus, $\bar{\phi}$ can be rewritten as:

$$\begin{aligned}
&\bar{\phi} = \delta\phi_{clcok} + \delta\phi_c(p_{rec}) + \delta_u\phi + N \\
&\delta\phi_{clcok} \equiv \frac{1}{\lambda}(c\delta t_{rec} - c\delta t_{sv}) \\
&\delta\phi_c(p_{rec}) \equiv \frac{1}{\lambda}(T^{'} - \frac{I}{f^2} - \delta p_{sv}) \\
&\delta_u\phi \equiv \frac{1}{\lambda}(m + \upsilon)
\end{aligned} \tag{5.13}$$

Considering the single difference between each auxiliary station and the master station, one obtains:

$$\begin{aligned}
&\Delta\bar{\phi} = \Delta\delta\phi_{clcok} + \Delta\delta_c\phi + \Delta\delta_u\phi + \Delta N \\
&l \equiv \bar{\phi} \\
&\Delta r \equiv -(\Delta\delta_c\phi + \Delta\delta_u\phi) = Dr \\
&\Delta C = \Delta\delta\phi_{clcok} + \Delta N
\end{aligned} \tag{5.14}$$

where D is a single difference matrix, as defined in Appendix A. This equation holds true if r is defined to be:

$$r \equiv -(\delta_c \phi + \delta_u \phi) \tag{5.15}$$

Equation (5.14) can be rewritten as:

$$\begin{aligned} &\Delta l + \Delta r - \Delta C = 0 \\ &f(\Delta l) = \Delta l - \Delta C = -\Delta r \\ &f(\Delta \hat{l}) = f(\Delta l + \Delta r) = 0 \end{aligned} \tag{5.16}$$

Note that the single difference clock error $\Delta\delta\phi_{clcok}$ can be cancelled out by forming differences between satellites. The single difference ambiguity term ΔN may contain a bias but it is common to one pair of stations. Thus, the bias can be cancelled out by forming differences between satellites. So the term ΔN is only related to the double difference ambiguities $\Delta\nabla N$, which are assumed to be known. Therefore, ΔC can be regarded as a known constant. Then, one can easily verify that the variance of single-differenced measurements equals the variance of single-differenced residuals and the covariance between signals and single-differenced measurements equals that between signals and single-differenced residuals, as follows:

$$C_{\Delta l,\Delta l} = C_{\Delta r,\Delta r} \quad and \quad C_{S,\Delta l} = C_{S,\Delta r} \quad and \quad C_{\Delta l,S} = C_{\Delta r,S}$$

where l is the measurement and r is the residual. Substituting this into (5.10), a collocation solution for the network can be expressed as:

$$
\begin{aligned}
&B = [0_{n\times p} \quad \& \quad I_{n\times n}] \\
&C^{*-1} = \begin{pmatrix} C_{\Delta r_p,\Delta r_p} & C_{\Delta r_p,\Delta l_n} \\ C_{\Delta l_n,\Delta r_p} & C_{\Delta l_n,\Delta l_n} \end{pmatrix}^{-1} = \begin{pmatrix} C_{\Delta r_p,\Delta r_p} & C_{\Delta r_p,\Delta r_n} \\ C_{\Delta r_n,\Delta r_p} & C_{\Delta r_n,\Delta r_n} \end{pmatrix}^{-1} \\
&w = f(\Delta l_n) = -\Delta r_n \\
&\hat{\Delta r}^{*} = -C^{*}B^{T}[BC^{*}B^{T}]^{-1}w = \begin{bmatrix} C_{\Delta r_p,\Delta r_n} \\ C_{\Delta r_n,\Delta r_n} \end{bmatrix} C^{-1}_{\nabla r_n,\nabla r_n} \Delta r_n \\
&\hat{\Delta r}_p = C_{\Delta r_p,\Delta r_n} C^{-1}_{\Delta r_n,\Delta r_n} \Delta r_n
\end{aligned}
\tag{5.17}
$$

In this case, the ∇r_n would be the correction difference between the auxiliary station and the master station, and the ∇r_p would be the correction difference between the rover and the master station.

5.4.3 Covariance Transformation From C_r

Two assumptions were made about the vector r (Raquet 1998): (1) it is well described as a Gaussian random vector with covariance matrix C_r; and (2) it is a zero-mean vector (i.e. $E[r] = 0$). So the covariance matrix of r could be written as:

$$C_r \equiv E[(r - E[r])(r - E[r])^T] = E[rr^T]$$

Also, as shown in Appendix A, $\Delta r \equiv Dr$. This means that Δr is a linear combination of r. So, the covariance matrix of the single difference errors ($C_{\Delta r,\Delta r}$) is:

$$C_{\Delta r,\Delta r} \equiv E[(Dr)(Dr)^T] = DE(rr^T)D = DC_{r,r}D$$

A summary of the relations between the various residual vectors and single difference vectors is given in the following equations.

$$\Delta r^* = \begin{bmatrix} \Delta r_{p_1} \\ \vdots \\ \Delta r_{p_P} \\ \Delta r_{n_1} \\ \vdots \\ \Delta r_{n_N} \end{bmatrix} = D \begin{bmatrix} r_0 \\ r_{p_1} \\ \vdots \\ r_{p_P} \\ r_0 \\ r_{n_1} \\ \vdots \\ r_{n_N} \end{bmatrix} = \begin{bmatrix} D_p & 0 \\ 0 & D_n \end{bmatrix} \begin{bmatrix} r_p \\ r_n \end{bmatrix}$$

$$C_{r^*} = \begin{bmatrix} c_{0,0} & c_{0,p_1} & \cdots & c_{0,p_P} & c_{0,0} & c_{0,n_1} & \cdots & c_{0,n_N} \\ c_{p_1,0} & c_{p_1,p_1} & \cdots & c_{p_1,p_P} & c_{p_1,0} & c_{p_1,n_1} & \cdots & c_{P_1,n_N} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\ c_{p_P,0} & c_{p_P,p_1} & \cdots & c_{p_P,p_P} & c_{p_P,0} & c_{p_P,n_1} & \cdots & c_{P_p,n_N} \\ c_{0,0} & c_{0,p_1} & \cdots & c_{0,p_P} & c_{0,0} & c_{0,n_1} & \cdots & c_{0,n_N} \\ c_{n_1,0} & c_{n_1,p_1} & \cdots & c_{n_1,p_P} & c_{n_1,0} & c_{n_1,n_1} & \cdots & c_{n_1,n_N} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ c_{n_N,0} & c_{n_N,p_1} & \cdots & c_{n_N,p_P} & c_{n_N,0} & c_{n_N,n_1} & \cdots & c_{n_N,n_N} \end{bmatrix} \quad (5.18)$$

$$= \begin{bmatrix} C_{r_p,r_p} & C_{r_p,r_n} \\ C_{r_n,r_p} & C_{r_n,r_n} \end{bmatrix}$$

$$
\begin{aligned}
C_{\Delta r_n,\Delta r_n} &\equiv D_n C_{r_n,r_n} D_n^{\ T} \\
C_{\Delta r_n,\Delta r_p} &\equiv \begin{bmatrix}0 & D_n\end{bmatrix} C_{r^*} \begin{bmatrix} D_p^T \\ 0 \end{bmatrix} = D_n C_{r_n,r_p} D_p^T \\
C_{\Delta r_p,\Delta r_p} &\equiv D_p C_{r_p,r_p} D_p^{\ T}
\end{aligned}
\qquad (5.19)
$$

In the RTCM 3.0 scheme, the interpolation module, which is located at the rover receiver end and will only have one interpolation (prediction) point, i.e itself. Thus, P equals 1 in the above equation and D_1 equals $[1\ \ -1]$.

5.4.4 Covariance Function

The covariance matrix C_{r^*} is composed of individual elements, c_{ab}^i, that can be calculated by a covariance function. Herein a covariance function described in (Raquet 1998) was chosen since its validity has been demonstrated through his investigation. The equations are summarized as follows:

$$
c_{ab}^i = \begin{cases} \mu^2(\varepsilon)[f_{z_c}(p_a, p_a, p_0) + \sigma_{u_z}^2(rec_a)] & a = b \\ \mu^2(\varepsilon) f_{z_c}(p_a, p_b, p_0) & a \neq b \end{cases}
$$

$$
f_{z_c}(p_a, p_b, p_0) = \frac{\sigma_{C_z}^2(p_a, p_0) + \sigma_{C_z}^2(p_b, p_0) - \sigma_{C_z}^2(p_a, p_b)}{2}
$$

$$
\sigma_{C_z}^2(p_a, p_b) = c_1 d + c_2 d^2
$$

$$
\sigma_{u_z}^2(rec) = const
$$

$$
\mu(\varepsilon) = \frac{1}{\sin\varepsilon} + c_\mu (0.53 - \frac{\varepsilon}{180^o})^3
$$

where p_0 is located at the master station. The parameters for L1/L2 phase measurements are given by Raquet (1998) as:

$$c_1 = 1.1024 \times 10^{-3}\ (cycles^2 / km)$$
$$c_2 = 4.8766 \times 10^{-7}\ (cycles^2 / km^2)$$
$$c_\mu = 3.9393$$
$$\sigma_{u_z}^2 = 4.4273 \times 10^{-5}\ (cycles^2)$$

The covariance matrix C_{r^*} can be generated using the above parameters. By substituting the covariance matrix C_{r^*} into (5.17), the $\Delta\hat{r}_p$ at the prediction point (rover) can then be estimated.

Fortes (2002) shows that the estimates are not sensitive to the covariance function selected. This shall substantially minimize the computation load of the least-squares collocation method, because there is no need to re-compute the covariance function often to follow the error distribution in the region.

5.4.5 Collocation Interpolation Surface

From the same four points, an interpolation surface, as shown in Figure 5.4, is generated by the collocation method. Unlike the plane interpolation, there will be no discrepancies at the physical points regardless of the number of CDs that contribute to determining the surface (Lin 2005).

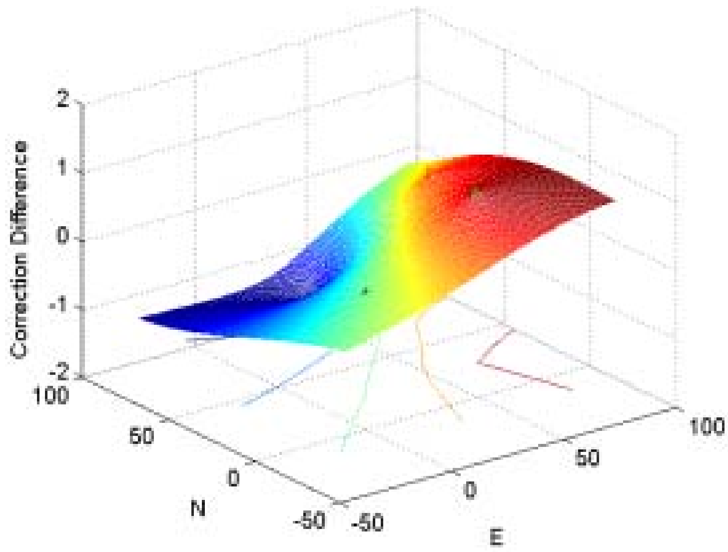

Figure 5.4: Least-squares collocation interpolation surface

Chapter Six: TEST RESULTS AND ANALYSIS

In this chapter, data from the Southern Alberta Network is used to analyze RTCM 3.0 implementation in a network RTK positioning software. Two days of data observed on May 24, 2004 and April 20, 2005 were selected for processing. Of these two datasets, one is under medium ionospheric activity while the other is under high ionospheric activity. The network corrections from these two datasets have distinct characteristics, in which the dataset on May 24, 2004 consists mostly of corrections computed from fix ambiguities and that on April 20, 2005 is a mix of corrections from both float and fix ambiguities. Analysis was carried out in the ambiguity domain, measurement domain, and position domain to evaluate the effectiveness of network approach under different ionospheric conditions. Also, a single baseline approach was used to compare with the network RTK approach in terms of position accuracy and ambiguity fix time.

6.1 Test Configuration

Figure 6.1 shows the entire post-mission test scheme. The network software, MutiRef™, reads observables from the network stations IRRI, COCH, STRA, BLDM and AIRD, then estimates the ambiguities between them and generates the RTCM 3.0 messages. As described in Section 5.1, a decoder and an interpolation function then interpret the messages and interpolate the corrections differences onto the position of the rover. The correction differences are applied back to the master station (IRRI) observables and a corrected observable RINEX file is generated to feed the reference station observables into the baseline processing software GrafNav™ 7.01. The UOFC observation files are also fed into GrafNav™ as the rover observables.

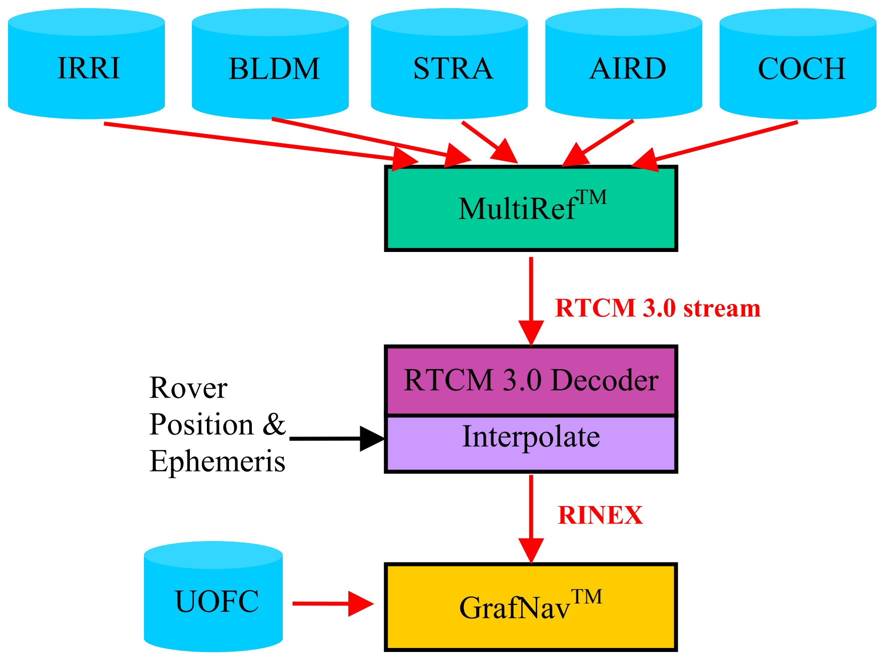

Figure 6.1: Post-mission test scheme

6.2 Methodology

The advantage of Network RTK positioning lies in its network corrections which can reduce the master-rover misclosures and therefore increase the speed and reliability of the ambiguity resolution (AR) at the rover end. However, the quality of network corrections mainly depends on the correctness of the ambiguity resolution within the network and how well the interpolation function fits the true error surface. Thus, several inter-related factors have been considered in this analysis.

First of all, the local K values, which indicate the severity of the ionospheric activity, are given to show the quality of the GPS observations. How well the network ambiguities are resolved depends on two main factors. One is the quality of input data – the GPS observations; the other is the efficiency of AR algorithms. As addressed in Liu (2003), the DD ionospheric and multipath-induced errors are the major error sources of the input observables which directly affect the performance of AR. Given that the physical stations in the network are selected deliberately so as to minimize their susceptibility to multipath, the effect of the latter may be safely considered to be negligible. Thus, the DD ionospheric errors become the key quality indicator associated with the GPS observations. Therefore the K values, which indicate the intensity of ionospheric activity, become the most important factors to be considered in selecting an appropriate period for analysis.

Secondly, the percentage of fixed ambiguities for each network baseline is another important quality indicator for the network corrections. Let us first recall equations (3.5) and (3.6), which give the definition of CD as the base for the following analysis:

$$
\begin{aligned}
L1CD_{AM}^{i} &= \frac{\Delta I_{AM}^{i}(t)}{f_1^2} - (\Delta T_{AM}^{i}(t) + \Delta \delta r_{AM}^{i}(t)) \\
L2CD_{AM}^{i} &= \frac{\Delta I_{AM}^{i}(t)}{f_2^2} - (\Delta T_{AM}^{i}(t) + \Delta \delta r_{AM}^{i}(t))
\end{aligned}
\tag{3.5}
$$

$$k_1 = \frac{f_2^2}{f_2^2 - f_1^2} = -1.545728, \quad k_2 = \frac{f_1^2}{f_1^2 - f_2^2} = 2.545728$$

$$\begin{aligned} ICPCD &= k_1 L1CD - k_1 L2CD \\ &= \frac{\Delta I(t)}{f_1^2} = \Delta I_1(t) \\ GCPCD &= k_2 L1CD + k_1 L2CD \\ &= \Delta T(t) + \Delta\delta r(t) \end{aligned} \tag{3.6}$$

Since RTK software will always form a double difference of the observations, only the double-differenced corrections are actually used in the baseline processing, although the single-differenced corrections (CD) are transmitted. The correction differences between satellites to get double-differenced corrections (DDC) can be further differenced as:

$$\begin{aligned} L1DDC_{AM}^{i,ref} &= \frac{\nabla\Delta I_{AM}^{i,ref}(t)}{f_1^2} - (\nabla\Delta T_{AM}^{i,ref}(t) + \nabla\Delta\delta r_{AM}^{i,ref}(t)) \\ L2DDC_{AM}^{i,ref} &= \frac{\nabla\Delta I_{AM}^{i,ref}(t)}{f_2^2} - (\nabla\Delta T_{AM}^{i,ref}(t) + \nabla\Delta\delta r_{AM}^{i,ref}(t)) \end{aligned} \tag{6.1}$$

$$\begin{aligned} ICPDDC_{AM}^{i,ref} &= \frac{\nabla\Delta I_{AM}^{i,ref}(t)}{f_1^2} = \nabla\Delta I_{AM\ 1}^{i,ref}(t) \\ GCPDDC_{AM}^{i,ref} &= \nabla\Delta T_{AM}^{i,ref}(t) + \Delta\delta r_{AM}^{i,ref}(t) \end{aligned} \tag{6.2}$$

where ref represents the base satellite.

However, these two equations hold true only when the DD ambiguities are fixed correctly and, in practice, this situation does not occur in every case. Sometimes, when the DD residuals are too large, the ambiguities may be unresolved or fixed to wrong values. Then Equations (6.1) and (6.2) should be modified to include the DD ambiguities errors as follows:

$$
\begin{aligned}
L1DDC &= \nabla\Delta\rho_{L1} - \frac{c}{f_1}\nabla\Delta\phi_{L1} + \frac{c}{f_1}\nabla\Delta\hat{N}_{L1} \\
&= \frac{\nabla\Delta I(t)}{f_1^2} - (\nabla\Delta T(t) + \nabla\Delta\delta r(t)) + \lambda_1\nabla\Delta\widetilde{N}_{L1} \\
L2DDC &= \nabla\Delta\rho_{L2} - \frac{c}{f_2}\nabla\Delta\phi_{L2} + \frac{c}{f_2}\nabla\Delta\hat{N}_{L2} \\
&= \frac{\nabla\Delta I(t)}{f_2^2} - (\nabla\Delta T(t) + \nabla\Delta\delta r(t)) + \lambda_2\nabla\Delta\widetilde{N}_{L2}
\end{aligned}
\tag{6.3}
$$

$$
\begin{aligned}
&k_1 = \frac{f_2^2}{f_2^2 - f_1^2} = -1.545728, \quad k_2 = \frac{f_1^2}{f_1^2 - f_2^2} = 2.545728 \\
&ICPDDC = \nabla\Delta I_1(t) + k_1(\lambda_1\nabla\Delta\widetilde{N}_{L1} - \lambda_2\nabla\Delta\widetilde{N}_{L2}) \\
&GCPDDC = (\nabla\Delta T(t) + \nabla\Delta\delta r(t)) + k_1(\lambda_2\nabla\Delta\widetilde{N}_{L2} - \frac{f_1^2}{f_2^2}\lambda_1\nabla\Delta\widetilde{N}_{L1})
\end{aligned}
\tag{6.4}
$$

where

$\nabla\Delta\hat{N}_{L1}$ and $\nabla\Delta\hat{N}_{L2}$: Estimated L1 and L2 DD ambiguities

$\nabla\Delta\widetilde{N}_{L1} = \nabla\Delta\hat{N}_{L1} - \nabla\Delta N_{L1}$: L1 DD ambiguities errors

$\nabla\Delta\widetilde{N}_{L2} = \nabla\Delta\hat{N}_{L2} - \nabla\Delta N_{L2}$: L2 DD ambiguities errors

Note that the superscript for the pair of satellites (i and ref) being differenced and the subscript for the pair of stations (one auxiliary station, A, and master station, M) being differenced are omitted.

It is obvious that the unknown DD ambiguity errors will cause the double-differenced corrections of each auxiliary station to depart from their true values. The corrections generated from float ambiguities will be denoted as “float” corrections in the following discussion. Obviously, the “float” corrections do not reflect the true errors (ionospheric, tropospheric and orbital errors) at the reference station point. Therefore the trustworthiness of interpolated corrections using these “float” corrections is questionable.

In the following analysis, the percentage of fixed ambiguities over all ambiguities occurring in a certain period is given for each baseline. The base satellites, fixed satellites and float satellites of each baseline are shown over the period of interest to give an approximate overview of testing conditions and, implicitly, the quality of the network corrections. Two sets of data were used to analyze the validity of using “float” corrections. In the first set, the network ambiguities are almost all resolved. The fix percentages of all the baselines are above 90%. However, in the second set, many network ambiguities are not resolved, in which one baseline is under 20% and two baselines are under 70%. For these two data sets, three interpolation methods are used to interpolate the corrections. Also, similar investigations are performed in observation and position domain to evaluate the “float” corrections and compared them with the “fixed” corrections.

Third, the DD misclosures between the raw master station IRRI and rover UOFC are compared with the misclosures between the corrected master station IRRI and rover UOFC. Because network corrections are intended to minimize the magnitude of the master-rover misclosures, the magnitude of misclosures with and without correction will be a direct quality indicator of the network corrections (e.g. Luo *et al* 2005). The DD misclosures can be written as:

$$
\begin{aligned}
Misc_{L1} &= \frac{\nabla\Delta I_{RM}^{ij}(t)}{f_1^2} - (\nabla\Delta T_{RM}^{ij}(t) + \nabla\Delta\delta r_{RM}^{ij}(t)) \\
Misc_{L2} &= \frac{\nabla\Delta I_{RM}^{ij}(t)}{f_2^2} - (\nabla\Delta T_{RM}^{ij}(t) + \nabla\Delta\delta r_{RM}^{ij}(t))
\end{aligned}
\qquad (6.5)
$$

Comparing the above equation with Equation (6.1), they are identical except that, in Equation (6.5), the misclosures are computed using the master station M and rover R, while, in Equation (6.1), the corrections are computed using two reference stations – the master station M and the auxiliary station A. In actuality, the corrections as described above are between-station misclosures.

Similarly, DD misclosures can be separated into two components, the dispersive and non-dispersive, by linear combination as following:

$$
\begin{aligned}
Disp\_Misc &= k_1 \times (Misc_{L1} - Misc_{L2}) \\
&= \nabla\Delta I_{1AM}^{ij}(t) \\
NonDisp\_Misc &= k_2 \times Misc_{L1} + k_1 \times Misc_{L2} \\
&= \nabla\Delta T_{AM}^{ij}(t) + \nabla\Delta\delta r_{AM}^{ij}(t)
\end{aligned}
\qquad (6.6)
$$

In the case that the DD ambiguities are either float or fixed, the DD misclosures will be affected by the DD ambiguity uncertainties, in a form similar to that implied by Equations (6.3) and (6.4). The misclosures computed from the float DD ambiguities are indicated as "float" misclosures. Likewise, "fix" misclosures are defined as misclosures of fixed satellites. Rewriting equations (6.5) and (6.6) using the estimated DD ambiguities $\nabla\Delta\hat{N}_{L1}$ and $\nabla\Delta\hat{N}_{L2}$ yields the following.

$$
\begin{aligned}
Misc_{L1} &= \nabla\Delta\rho_{L1} - \frac{c}{f_1}\nabla\Delta\phi_{L1} + \frac{c}{f_1}\nabla\Delta\hat{N}_{L1} \\
&= \frac{\nabla\Delta I(t)}{f_1^2} - (\nabla\Delta T(t) + \nabla\Delta\delta r(t)) + \frac{c}{f_1}(\nabla\Delta\hat{N}_{L1} - \nabla\Delta N_{L1}) \\
Misc_{L2} &= \nabla\Delta\rho_{L2} - \frac{c}{f_2}\nabla\Delta\phi_{L2} + \frac{c}{f_2}\nabla\Delta\hat{N}_{L2} \\
&= \frac{\nabla\Delta I(t)}{f_2^2} - (\nabla\Delta T(t) + \nabla\Delta\delta r(t)) + \frac{c}{f_2}(\nabla\Delta\hat{N}_{L2} - \nabla\Delta N_{L2})
\end{aligned} \tag{6.7}
$$

Define $\nabla\Delta\tilde{N}_{L1} = \nabla\Delta\hat{N}_{L1} - \nabla\Delta N_{L1}, \quad \nabla\Delta\tilde{N}_{L2} = \nabla\Delta\hat{N}_{L2} - \nabla\Delta N_{L2}$

$$
\begin{aligned}
&Disp\_Misc = \nabla\Delta I_1(t) + k_1(\lambda_1\nabla\Delta\tilde{N}_{L1} - \lambda_2\nabla\Delta\tilde{N}_{L2}) \\
&NonDisp\_Misc = (\nabla\Delta T(t) + \nabla\Delta\delta r(t)) + k_1(\lambda_2\nabla\Delta\tilde{N}_{L2} - \frac{f_1^2}{f_2^2}\lambda_1\nabla\Delta\tilde{N}_{L1})
\end{aligned} \tag{6.8}
$$

In this analysis, equations (6.7) and (6.8) can be seen as representations of the misclosures of the raw observations of the master station IRRI and the rover UOFC, which can be computed by rerunning MultiRef^{TM} by fixing the coordinates of UOFC. In the following, the misclosures generated from the raw observations will be referred to as "raw" misclosures. As shown in Section 5.1, a corrected IRRI RINEX observation file will be generated by applying network corrections onto the raw observations of IRRI. It is expected that, after the corrections are applied, the DD misclosures (i.e., errors) of IRRI-UOFC will decrease. Similarly, the so-called "corrected" misclosures can be computed by running MultiRef™ again using the corrected master station observations and raw observations of UOFC. Using $\nabla\Delta C_{L1}, \nabla\Delta C_{L2}$ to denote the DD corrections to be applied onto the master station observations, the "corrected" misclosures can be represented in accordance with following two equations:

$$
\begin{aligned}
Misc_{L1}{}^{c} &= \nabla\Delta\rho_{L1} - \lambda_1(\nabla\Delta\phi_{L1} + \nabla\Delta C_{L1}) + \lambda_1\nabla\Delta\hat{N}_{L1}{}^{c} \\
&= \frac{\nabla\Delta I(t)}{f_1^{2}} - (\nabla\Delta T(t) + \nabla\Delta\delta r(t)) + \lambda_1(\nabla\Delta\hat{N}_{L1}{}^{c} - \nabla\Delta N_{L1}) - \lambda_1\nabla\Delta C_{L1} \\
Misc_{L2}{}^{c} &= \nabla\Delta\rho_{L2} - \lambda_2(\nabla\Delta\phi_{L2} + \nabla\Delta C_{L2}) + \lambda_2\nabla\Delta\hat{N}_{L2}{}^{c} \\
&= \frac{\nabla\Delta I(t)}{f_2^{2}} - (\nabla\Delta T(t) + \nabla\Delta\delta r(t)) + \lambda_2(\nabla\Delta\hat{N}_{L2}{}^{c} - \nabla\Delta N_{L2}) - \lambda_2\nabla\Delta C_{L2}
\end{aligned}
\tag{6.9}
$$

Define $\nabla\Delta\tilde{N}_{L1}{}^{c} = \nabla\Delta\hat{N}_{L1}{}^{c} - \nabla\Delta N_{L1}, \quad \nabla\Delta\tilde{N}_{L2}{}^{c} = \nabla\Delta\hat{N}_{L2}{}^{c} - \nabla\Delta N_{L2}$

$$\nabla\Delta C_I = k_1(\nabla\Delta C_{L1} - \nabla\Delta C_{L2}), \quad \nabla\Delta C_G = k_1\nabla\Delta C_{L1} + k_2\nabla\Delta C_{L2}$$

$$
\begin{aligned}
Disp\_Misc^{c} &= \nabla\Delta I_1(t) + k_1(\lambda_1\nabla\Delta\tilde{N}_{L1}{}^{c} - \lambda_2\nabla\Delta\tilde{N}_{L2}{}^{c}) - \nabla\Delta C_I \\
NonDisp\_Misc^{c} &= (\nabla\Delta T(t) + \nabla\Delta\delta r(t)) + k_1(\lambda_2\nabla\Delta\tilde{N}_{L2}{}^{c} - \frac{f_1^2}{f_2^2}\lambda_1\nabla\Delta\tilde{N}_{L1}{}^{c}) - \nabla\Delta C_G
\end{aligned}
\tag{6.10}
$$

It is important to remember that the DD ambiguities should be the same regardless of whether the raw observations of the master station or the corrected observations are used. This is because only when there is a constant integer cycle bias in the DD corrections will the true ambiguities be altered. However, by their very nature, DD corrections should not include a constant integer cycle bias. That is why same terms $\nabla\Delta N_{L1}$ and $\nabla\Delta N_{L2}$ are used to represent the true DD ambiguities in Equation (6.7) and (6.9). Nevertheless, the estimated ambiguities could be different using raw and corrected observations. In another word, $\nabla\Delta\hat{N}_{L1}{}^{c}$ and $\nabla\Delta\hat{N}_{L2}{}^{c}$, which represent respectively the estimated L1 and L2 DD ambiguities using the corrected master station and the rover observations, could be different from the $\nabla\Delta\hat{N}_{L1}$ and $\nabla\Delta\hat{N}_{L2}$ estimated ambiguities using the raw observations of master station. However, when the estimated ambiguities $\nabla\Delta\hat{N}_{L*}$ and $\nabla\Delta\hat{N}_{L*}{}^{c}$ are correctly fixed, they are identical. i.e $\nabla\Delta\hat{N}_{L1} = \nabla\Delta\hat{N}_{L1}{}^{c} = \nabla\Delta N_{L1}$ or $\nabla\Delta\hat{N}_{L2} = \nabla\Delta\hat{N}_{L2}{}^{c} = \nabla\Delta N_{L2}$.

As discussed in Sections 3.3 and 3.4, the corrections are supposed to cancel out the DD ionospheric, tropospheric and orbital errors, therefore decrease the dispersive and non-dispersive misclosures. However, the unknowns, consisting of the estimated ambiguity errors $\nabla\Delta\tilde{N}_{L*}$ and $\nabla\Delta\tilde{N}_{L*}{}^{c}$ hinder the perceptibility of benefits from corrections. This can be explained through an exploration of the following four cases.

Case 1: $\nabla\Delta\tilde{N}_{L*} = 0$ and $\nabla\Delta\tilde{N}_{L*}{}^{c} = 0$

The first case indicates that the estimated ambiguities are fixed correctly to the true ambiguities. Equations (6.4) and (6.6) can, therefore, be rewritten as:

$$\begin{aligned} &Disp\_Misc = \nabla\Delta I_1(t) \\ &NonDisp\_Misc = (\nabla\Delta T(t) + \nabla\Delta\delta r(t)) \end{aligned} \qquad (6.11)$$

$$Disp\_Misc^c = \nabla\Delta I_1(t) - \nabla\Delta C_I$$
$$NonDisp\_Misc^c = (\nabla\Delta T(t) + \nabla\Delta\delta r(t)) - \nabla\Delta C_G \tag{6.12}$$

In this case, the effect of corrections can be readily understood by comparing the "raw" misclosures and "corrected" misclosures.

Case 2: $\nabla\Delta\widetilde{N}_{L*} \neq 0$ and $\nabla\Delta\widetilde{N}_{L*}{}^c = 0$

In the second case, the ambiguities estimated using raw observations are incorrectly attributed as either float or fixed, while the estimated ambiguities using the corrected observations are fixed correctly. Equation (6.4) remains the same and Equation (6.6) is reduced to the form previously given in Equation (6.8). Rewriting and condensing them yields the following:

$$Disp\_Misc = \nabla\Delta I_1(t) + k_1(\lambda_1\nabla\Delta\widetilde{N}_{L1} - \lambda_2\nabla\Delta\widetilde{N}_{L2})$$
$$NonDisp\_Misc = (\nabla\Delta T(t) + \nabla\Delta\delta r(t)) + k_1(\lambda_2\nabla\Delta\widetilde{N}_{L2} - \frac{f_1^2}{f_2^2}\lambda_1\nabla\Delta\widetilde{N}_{L1})$$

$$Disp\_Misc^c = \nabla\Delta I_1(t) - \nabla\Delta C_I$$
$$NonDisp\_Misc^c = (\nabla\Delta T(t) + \nabla\Delta\delta r(t)) - \nabla\Delta C_G$$

It is well known that the float ambiguities will absorb systematic errors and reduce the magnitude of misclosures. In this case, a direct comparison of the "raw" misclosures and "corrected" misclosures could be very risky, since the unknown ambiguity errors could possibly reduce the magnitude of misclosures more than the corrections do. In fact, this situation means that network corrections benefit the baseline processing, because the ambiguities can be resolved correctly after corrections are applied.

Case 3: $\nabla\Delta\widetilde{N}_{L*} = 0$ and $\nabla\Delta\widetilde{N}_{L*}{}^{c} \neq 0$

In this case, the estimated ambiguities deduced with the use of corrected observations are incorrectly attributed as float or fixed, while the estimated ambiguities using raw observations are fixed correctly. Equation (6.6) remains the same, while Equation (6.4) is reduced to the form shown above as Equation (6.7). Rewriting these yields the following:

$$Disp\_Misc = \nabla\Delta I_1(t)$$
$$NonDisp\_Misc = (\nabla\Delta T(t) + \nabla\Delta\delta r(t))$$

$$Disp\_Misc^{c} = \nabla\Delta I_1(t) + k_1(\lambda_1\nabla\Delta\widetilde{N}_{L1}{}^{c} - \lambda_2\nabla\Delta\widetilde{N}_{L2}{}^{c}) - \nabla\Delta C_I$$
$$NonDisp\_Misc^{c} = (\nabla\Delta T(t) + \nabla\Delta\delta r(t)) + k_1(\lambda_2\nabla\Delta\widetilde{N}_{L2}{}^{c} - \frac{f_1^2}{f_2^2}\lambda_1\nabla\Delta\widetilde{N}_{L1}{}^{c}) - \nabla\Delta C_G$$

In this case, it is difficult to isolate the effects of ambiguity errors and corrections. Direct comparison of the "raw" misclosures and "corrected" misclosures remains very risky at this point. Furthermore, it is possible that the magnitude of the "corrected" misclosures is smaller than the "raw" misclosures. Actually, network corrections worsen the performance since the ambiguities are correctly resolved before applying corrections but they cannot be resolved correctly after corrections are applied.

Case 4: $\nabla\Delta\widetilde{N}_{L*} \neq 0$ and $\nabla\Delta\widetilde{N}_{L*}{}^{c} \neq 0$

This condition implies that the ambiguities estimated with the use of corrected observations or raw observations are attributed as float or fixed incorrectly. Equations (6.4) and (6.6) remain the same. These can be rewritten in the following form:

$$Disp\_Misc = \nabla\Delta I_1(t) + k_1(\lambda_1\nabla\Delta\widetilde{N}_{L1} - \lambda_2\nabla\Delta\widetilde{N}_{L2})$$
$$NonDisp\_Misc = (\nabla\Delta T(t) + \nabla\Delta\delta r(t)) + k_1(\lambda_2\nabla\Delta\widetilde{N}_{L2} - \frac{f_1^2}{f_2^2}\lambda_1\nabla\Delta\widetilde{N}_{L1})$$

$$Disp\_Misc^c = \nabla\Delta I_1(t) + k_1(\lambda_1\nabla\Delta\widetilde{N}_{L1}{}^c - \lambda_2\nabla\Delta\widetilde{N}_{L2}{}^c) - \nabla\Delta C_I$$
$$NonDisp\_Misc^c = (\nabla\Delta T(t) + \nabla\Delta\delta r(t)) + k_1(\lambda_2\nabla\Delta\widetilde{N}_{L2}{}^c - \frac{f_1^2}{f_2^2}\lambda_1\nabla\Delta\widetilde{N}_{L1}{}^c) - \nabla\Delta C_G$$

In this case, the comparison of "raw" and " corrected" misclosures becomes meaningless since the effects of ambiguity errors and corrections are interwoven with each other.

To summarize, the corrections are supposed to cancel out the DD ionospheric, tropospheric and orbital errors, which herein decrease the dispersive and non-dispersive misclosures. However, in practice, because it is impossible to determine the DD ambiguities in every instance, the estimated ambiguity errors $\nabla\Delta\widetilde{N}_{L*}$ and $\nabla\Delta\widetilde{N}_{L*}{}^c$ remain encapsulated as non-distinct constituents of the misclosures. This therefore blurs the effects of corrections, indicating that the corrections can be traced only when $\nabla\Delta\widetilde{N}_{L*} = 0$ and $\nabla\Delta\widetilde{N}_{L*}{}^c = 0$.

To simplify the calculation and to isolate the effects of $\nabla\Delta\widetilde{N}_{L*}{}^c$, the "corrected" misclosures are computed by applying the corrections directly onto the "raw" misclosures instead of rerunning MutiRef™ using the corrected master station. The "corrected" misclosures can therefore be represented as:

$$
\begin{aligned}
Disp\_Misc^{c} &= \nabla\Delta I_1(t) + k_1(\lambda_1 \nabla\Delta\tilde{N}_{L1} - \lambda_2 \nabla\Delta\tilde{N}_{L2}) - \nabla\Delta C_I \\
NonDisp\_Misc^{c} &= (\nabla\Delta T(t) + \nabla\Delta\delta r(t)) + k_1(\lambda_2 \nabla\Delta\tilde{N}_{L2} - \frac{f_1^2}{f_2^2}\lambda_1 \nabla\Delta\tilde{N}_{L1}) - \nabla\Delta C_G
\end{aligned} \tag{6.13}
$$

In this way, when $\nabla\Delta\tilde{N}_{L*} = 0$ (i.e., the DD ambiguities are fixed correctly using the raw measurements), this method is equivalent to the last method outlined in Case 1 above. This allows the efficiency of the corrections to be displayed directly from the "raw" and "corrected" misclosures.

Lastly, the analysis is carried out in the position domain. Three network RTK results using different interpolation methods; i.e. distance weighted, plane and collocation, respectively, are given. Also, for purposes of comparison, a single reference station RTK result is also presented. In the single reference station RTK processing, AIRD, which is the nearest station to UOFC, is chosen as the reference station and GravNav™ 7.01 is also used for the baseline processing.

The test and analysis procedures can be summarized as follows:

First, the network software MultiRef™ was run with the raw observables at IRRI, COCH, STRA, BLDM and AIRD, and try to fix the ambiguities of each pair of stations and satellites. Simultaneously the RTCM 3.0 messages were generated and saved in a file. The ambiguity status of each baseline and each pair of satellites will be presented.

Next, MultiRef^{TM} is rerun using the raw observables of IRRI and UOFC to obtain the DD misclosures on a fixed baseline. Then, the interpolated corrections are applied to the misclosures to display the efficiency of corrections by directly comparing the "raw" and "corrected" misclosures. For an un-resolved baseline, the "float" misclosures will also be presented. However, corrections will not be applied to the misclosures since, as shown above, the comparison of "raw float" misclosures and "corrected float" misclosures cannot reflect the correctness and, hence, the efficiency of corrections. Therefore, the corrections will be presented, but not applied to the "float" misclosures.

Last, the two sets of observables - i.e., the corrected master station IRRI observables (using three interpolation methods) and the raw AIRD observables - are processed by GravNav^{TM} as master stations to compute the position of rover UOFC.

It should be noticed that because GravNav^{TM} cannot output misclosures, MultiRef^{TM} has to be used to calculate the misclosures of the baseline IRRI-UOFC and to do the analysis in the observation domain. But GravNav^{TM} is used for analysis in the position domain. Therefore, the ambiguity resolution status using GravNav^{TM} given in Section 6.4.3 and 6.5.3 may not match the status using MultiRef^{TM} given in Section 6.4.2 and 6.5.2.

6.3 Data Selection

In order to obtain an approximate understanding of the ionospheric activity in the network, local ionospheric K values are retrieved from the MEANOOK magnetometer station, which is located in Edmonton about 300 kilometres north of the UOFC station. These values, calculated on the basis of observations of magnetic field fluctuations, range from 1 to 9, representing the range of quiet to extreme ionospheric activity. The K values on May 24, 2004 and Apr 20, 2005 are shown in Table 6.1.

Table 6.1: K values on May 24, 2004 and Apr 20, 2005

UTC time	0-3	3-6	6-9	9-12	12-15	15-18	18-21	21-24
Local time	17-20	20-23	23-2	2-5	5-8	8-11	11-14	14-17
May 24, 2004	3	1	4	3	4	2	2	2
Apr 20, 2005	2	5	6	6	4	2	2	2

On May 24, 2004, ionospheric activity was at medium intensity during the night-time, and local K values of 3 to 4 were observed. On the evening of April 20, 2005, local K values of 5 to 6 were observed in the period from 20:00 to 5:00 local time on the 21st, indicating a more active ionosphere. Quiet ionospheric conditions were experienced during the daytime hours on both days, with local K values of 2.

In the following two test scenarios, MutiRef™ began running at 17:00 local time (00:00 UTC), which means that the RTCM 3.0 messages were generated from 00:00 until 24:00 UTC. Consequently the intermediate output - corrected master station observation file started at 00:00 UTC and ended at 24:00 UTC. However, GrafNav™ began its processing at different times to facilitate analysis. MutiRef™ and GrafNav™ did not run synchronously because MutiRef™ needs processing time to resolve the ambiguities or, alternatively, to achieve convergence of the float ambiguity solution. An additional logistical issue is the service provider's preference to initialize the network software in a favorable (i.e., quiet ionosphere) environment. However, unlike the network software, the rover could start working at any time. In the following tests, the GrafNav™ started to run at a specific time to process a certain period.

6.4 Test Scenario I

From Table 6.1, the K values range between 3 and 4 from local time 1:00 to 3:00 on May 24, 2004. Therefore, this period of time was selected for an analysis of the way in which network corrections affect the misclosures between the rover and the master station IRRI. Analysis is also carried out in the position domain to compare network RTK positioning performance using three different interpolation methods and a single station RTK approach.

The network configuration described in Section 4.5, is shown again in Figure 6.2

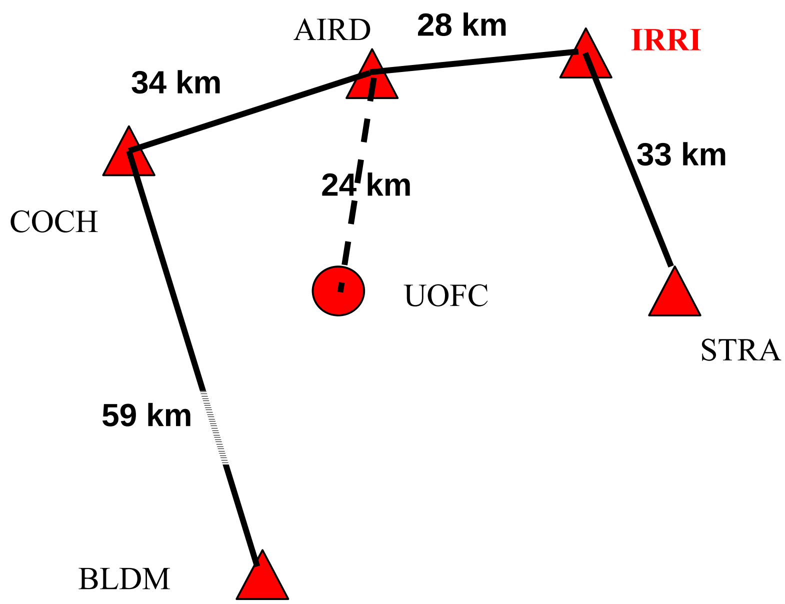

Figure 6.2: Network configuration (the master station is shown in red.)

6.4.1 Ambiguity Resolution Status

Based on the shortest baseline rule incorporated into the processing routines of MultRef™, the program attempts to solve ambiguities between inter-connected stations, as shown in Figure 6.2. Figure 6.3 presents the L1 and L2 ambiguity status of the four baselines in the network for local time 01:00 to 03:00 May 25, 2004. The green points denote the base satellites, the blue ones denote the fixed satellites, and the red ones represent satellites with float ambiguities.

From these figures and the statistics given in Table 6.2, one can see that the percentages of fixed ambiguities are very high and above 90%. Especially in the case of the baseline

IRRI-AIRD, almost all the ambiguities are fixed. This scenario is a good example of a case where network ambiguities are resolved at a high level.

Table 6.2: Percentage of fixed ambiguities

Baseline	**IRRI-STRA**	**IRRI-AIRD**	**COCH-BLDM**	**COCH-AIRD**
L1 %	91.57	99.84	94.32	97.22
L2 %	91.57	99.84	93.21	96.24

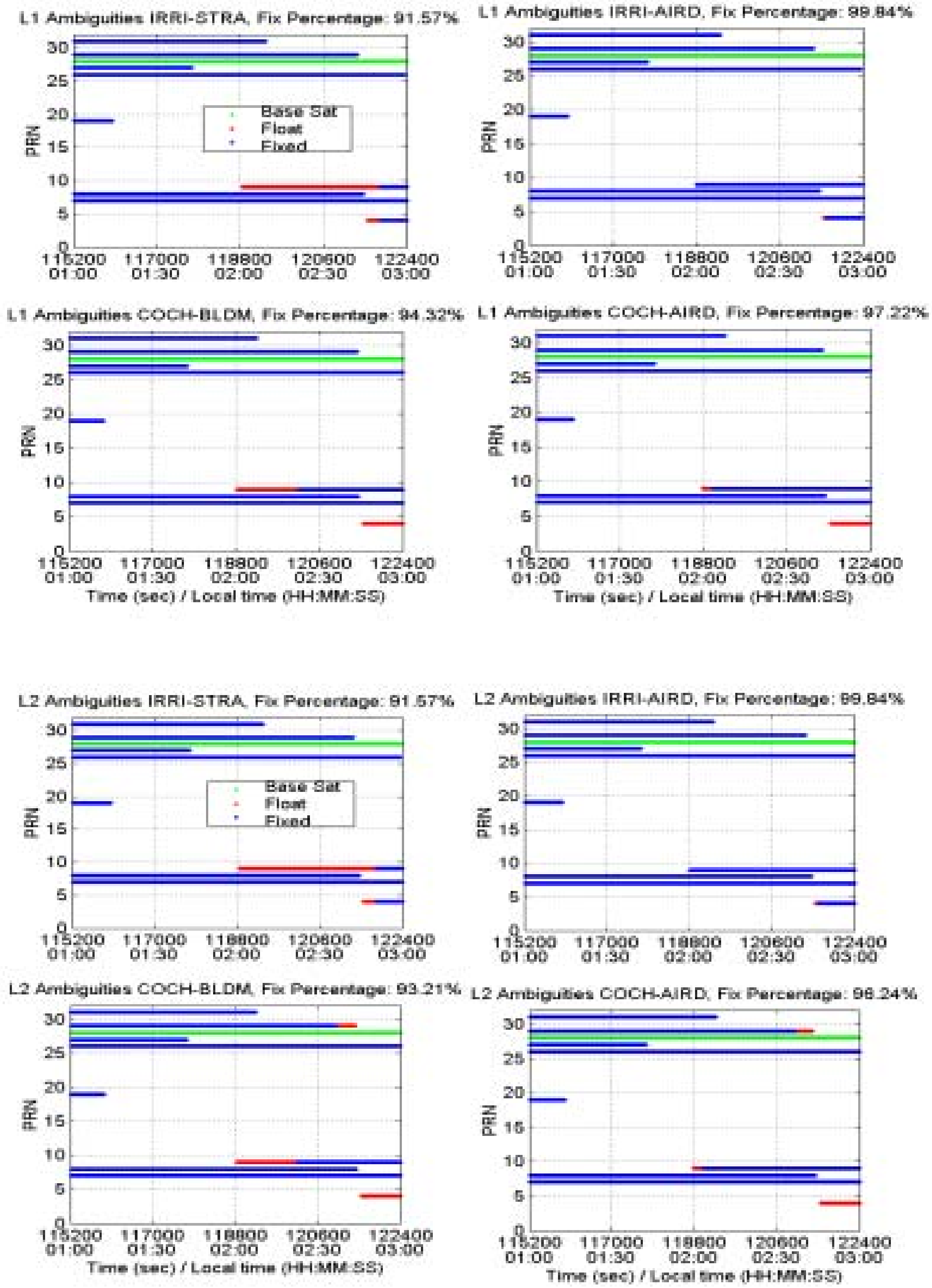

Figure 6.3: L1 & L2 ambiguity resolution status in the network

6.4.2 Observation Domain

MultiRef™ was rerun using the raw observables of IRRI and UOFC. The status of L1 and L2 ambiguities, respectively, is shown in Figure 6.4. In this case, the average percentage of fixed ambiguities is 92.63% for L1 and 91.77% for L2. For L1, PRN 9 took more than 45 minutes to fix and PRN 4 about one minute, while the other satellites were fixed all the time. For L2, PRN 9 also took around 45 minutes to fix, PRN 4 about one minute, and additionally PRN 31 lost fix towards the end of its availability period. All the other satellites were fixed throughout the period.

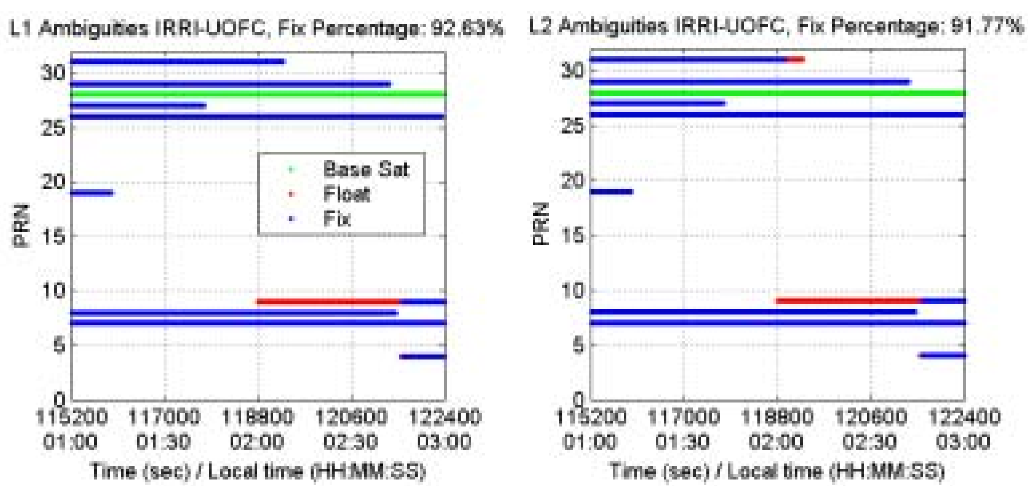

Figure 6.4: L1 and L2 ambiguities resolution between IRRI and UOFC

For all of the fixed satellites, Figure 6.5 presents their dispersive “raw” and “corrected” misclosures using distance weighted, plane and collocation interpolation methods, respectively. All three interpolation methods achieve significant improvement for the dispersive component - generally, over 60% can be observed as shown in Table 6.3. In this dataset, the collocation method achieved the best results in terms of percentage of improvement, followed by the plane and distance weighted methods. Table 6.3 gives the percentage of improvement of all the satellites after applying the network corrections. Except for PRN 19, the collocation method produced a 70% to 80% improvement of all the other fixed satellites; the plane method, around 70% and the distance weighted method, around 60%. After applying network corrections, the magnitude of dispersive misclosures of all satellites is successfully lowered to less than 0.3 L1 cycles.

However, the non-dispersive misclosures, which have been reduced by the tropospheric model, have very small magnitudes. As shown in Figure 6.6, the magnitude of non-dispersive misclosures is normally less than 0.5 L1 cycles. Comparing Figure 6.6 and Figure 6.5, it is clear that the improvement in non-dispersive misclosures after correction is not as obvious as the effect on dispersive misclosures. However, from Table 6.4 which shows the RMS of non-dispersive "raw" and "corrected" misclosures, it is apparent that a reasonable improvement, about 10% to 30 %, can still be obtained through the application of corrections.

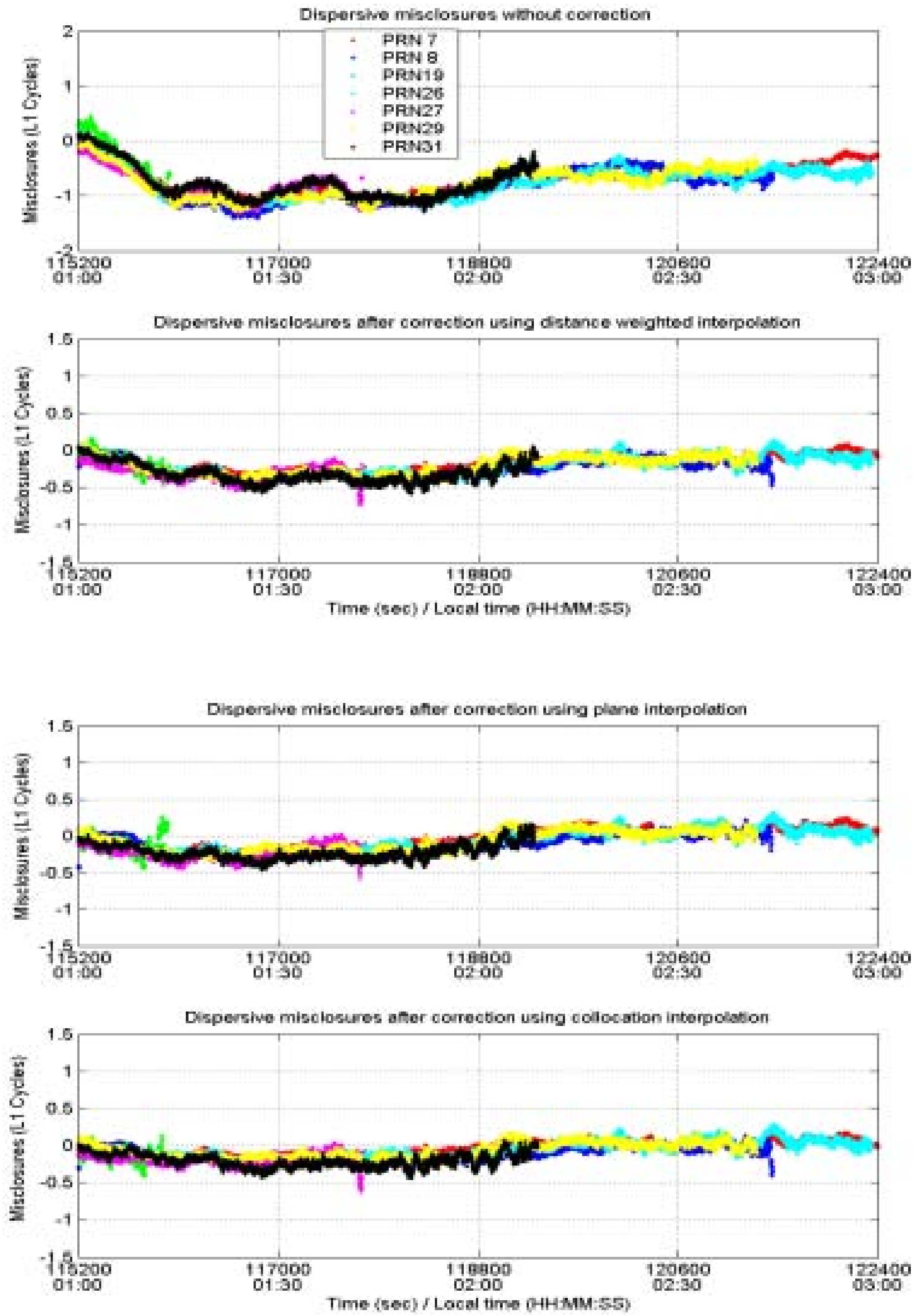

Figure 6.5: Dispersive “raw” and “corrected” misclosures using distance weighted, plane and collocation interpolation methods

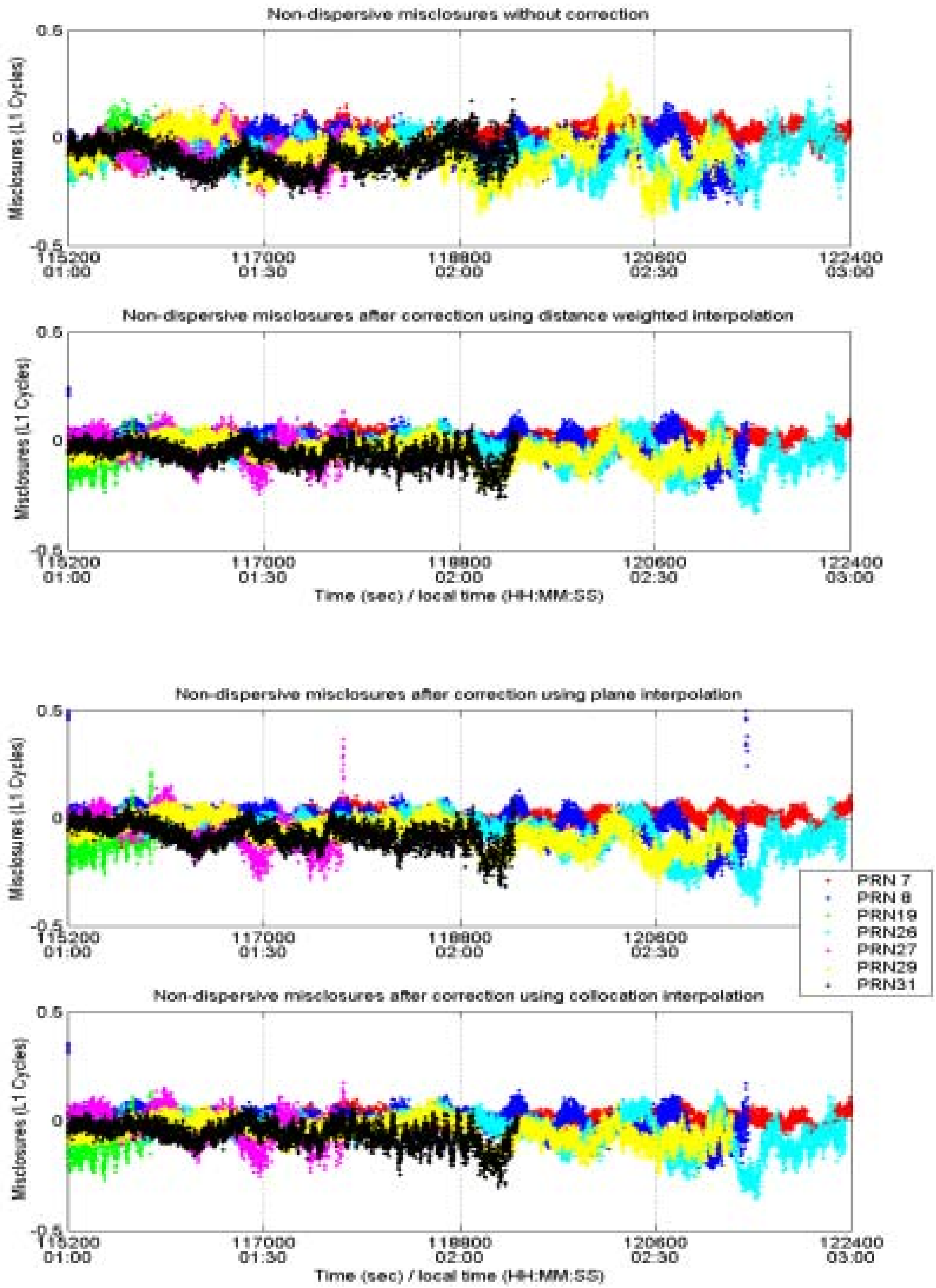

Figure 6.6: Non-dispersive "raw" and "corrected" misclosures using distance weighted, plane and collocation interpolation methods

Table 6.3: RMS of "raw" and "corrected" dispersive misclosures in L1 and improvement with corrections

	Without correction	**Distance weighted**		**Plane**		**Collocation**	
PRN	**RMS (L1 cycle)**	**RMS (L1 cycle)**	**Imp.**	**RMS (L1 cycle)**	**Imp.**	**RMS (L1 cycle)**	**Imp.**
7	0.7527	0.210	62%	0.1484	69%	0.1026	74%
8	0.8660	0.2785	67%	0.1719	79%	0.1373	83%
19	0.4925	0.2032	33%	0.1799	35%	0.1587	38%
26	0.8159	0.2365	66%	0.1508	76%	0.1162	80%
27	0.8737	0.3271	63%	0.2540	70%	0.2028	76%
29	0.8248	0.2461	66%	0.1643	75%	0.1213	80%
31	0.8407	0.3393	57%	0.2495	67%	0.222	70%

Table 6.4: RMS of "raw" and "corrected" non-dispersive misclosures in L1 cycle and improvement with corrections

PRN	**Without correction**	**Distance weighted**		**Plane**		**Collocation**	
	RMS (L1 cycle)	**RMS (L1 cycle)**	**Imp.**	**RMS (L1 cycle)**	**Imp.**	**RMS (L1 cycle)**	**Imp.**
7	0.0469	0.0322	12%	0.0376	8%	0.0316	13%
8	0.0716	0.0493	19%	0.0673	4%	0.0522	16%
19	0.0806	0.0971	14%	0.1244	37%	0.1127	27%
26	0.1061	0.0782	23%	0.1009	4%	0.0822	20%
27	0.0945	0.0688	21%	0.0952	1%	0.0734	18%
29	0.1186	0.0693	41%	0.0931	21%	0.0698	41%
31	0.1050	0.0773	23%	0.0946	9%	0.0795	22%

The following analysis focuses on PRN 4, whose ambiguity is not resolved at the beginning of the observation period, and become fixed after several minutes, as can be seen in Figure 6.4. Figure 6.7 to Figure 6.9 contain the descriptive results of this analysis.

Figure 6.7 presents the L1 and L2 ambiguities of PRN 4 – PRN28 for each baseline respectively, in which PRN28 is the base satellite. Also to provide a better interpretation of the ambiguities, they were moved to around zero by subtracting their nearest integers, which are also shown in the lower right hand side of each plot. But only the "float" and "fixed" characteristics of ambiguities matter in the analysis. As shown in this figure, the blue and green points represent the values of the float and fixed ambiguities, respectively. Only baselines IRRI-AIRD and IRRI-STRA are fixed partially over time, while baselines COCH-BLDM and COCH-AIRD are float throughout. Therefore, the corrections generated from auxiliary stations AIRD and STRA are partially float, and those generated from auxiliary stations BLDM and COCH are consistently float.

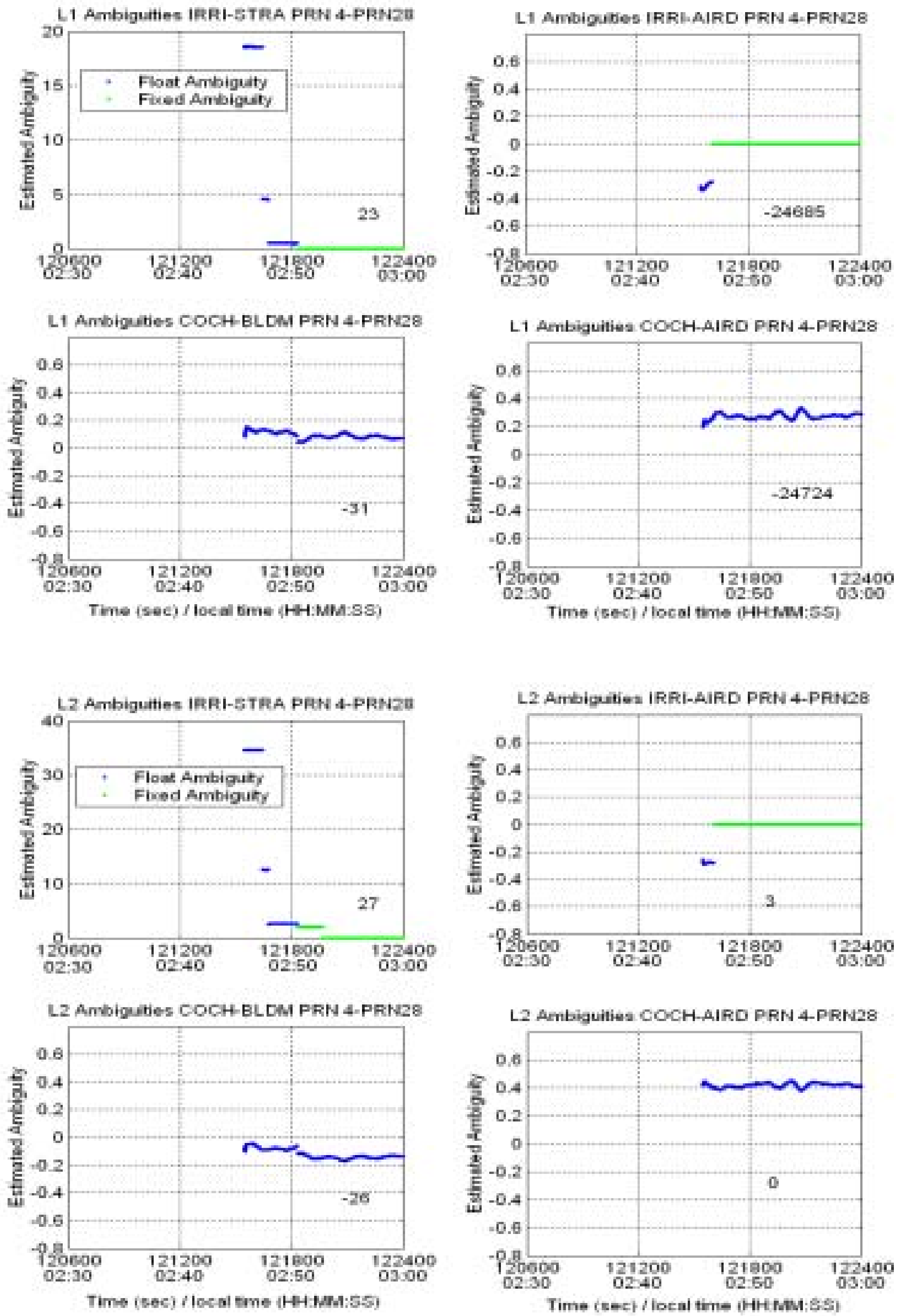

Figure 6.7: PRN 4 - PRN28 (base), L1 and L2 ambiguities of each baseline in the network

Figure 6.8 presents the PRN 4-PRN28, L1 and L2 ambiguities on baseline IRRI-UOFC. The L1 & L2 ambiguities are fixed after about one minute. Correspondingly, the "raw" misclosures of PRN4-PRN28 for the baseline IRRI-UOFC are float in the first minute and become fixed afterwards.

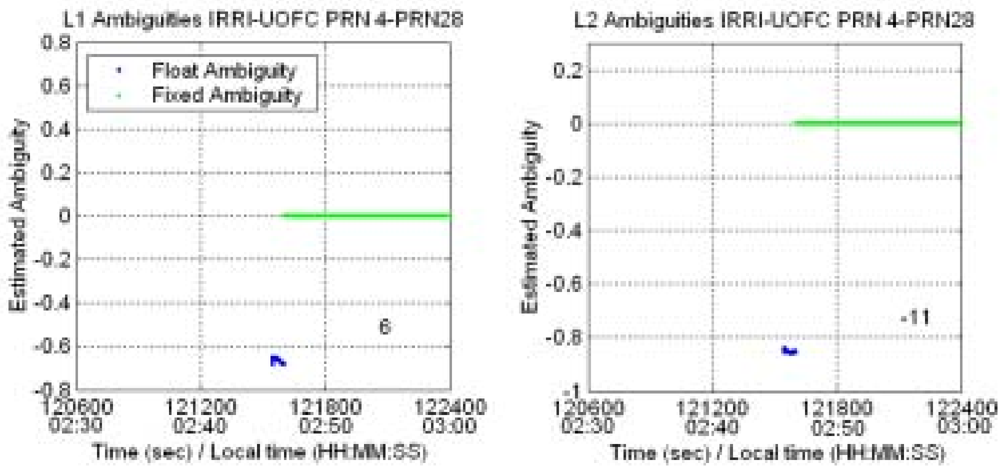

Figure 6.8: PRN4 – PRN28 (base), L1 and L2 ambiguities on baseline IRRI-UOFC

Figure 6.9 presents the "raw" misclosures and interpolated corrections generated from the RTCM 3.0 network corrections using three interpolation methods. According to the ambiguity status given in Figure 6.8, the ambiguities are initially float and then fixed.. This largely explains why a noticeable bias can be observed in the misclosures shown in Figure 6.9, a finding that is consistent with Equation (6.8), which is rewritten as follows:

$$Disp\_Misc = \nabla\Delta I_1(t) + k_1(\lambda_1 \nabla\Delta\tilde{N}_{L1} - \lambda_2 \nabla\Delta\tilde{N}_{L2})$$
$$NonDisp\_Misc = (\nabla\Delta T(t) + \nabla\Delta\delta r(t)) + k_1(\lambda_2 \nabla\Delta\tilde{N}_{L2} - \frac{f_1^2}{f_2^2}\lambda_1 \nabla\Delta\tilde{N}_{L1})$$

Ambiguity errors will induce errors in the misclosures. In another word, the misclosures computed from float ambiguities consist of true ionospheric and geometric errors, as well as ambiguity errors. Once the ambiguities are fixed correctly on both L1 and L2, the dispersive and non-dispersive misclosures will equal to the true ionospheric and geometric errors.

In addition, the interpolated dispersive corrections, regardless of which methods is employed, are typically around zero. Obviously, they cannot compensate for the true dispersive errors. As for the non-dispersive corrections, because the magnitude of true geometric errors is very small, it is difficult to make any related assertions. However, since the dispersive and non-dispersive corrections are all around zero, a conservative estimation can be made that the network corrections of PRN 4 do not bring much improvement in observation domain.

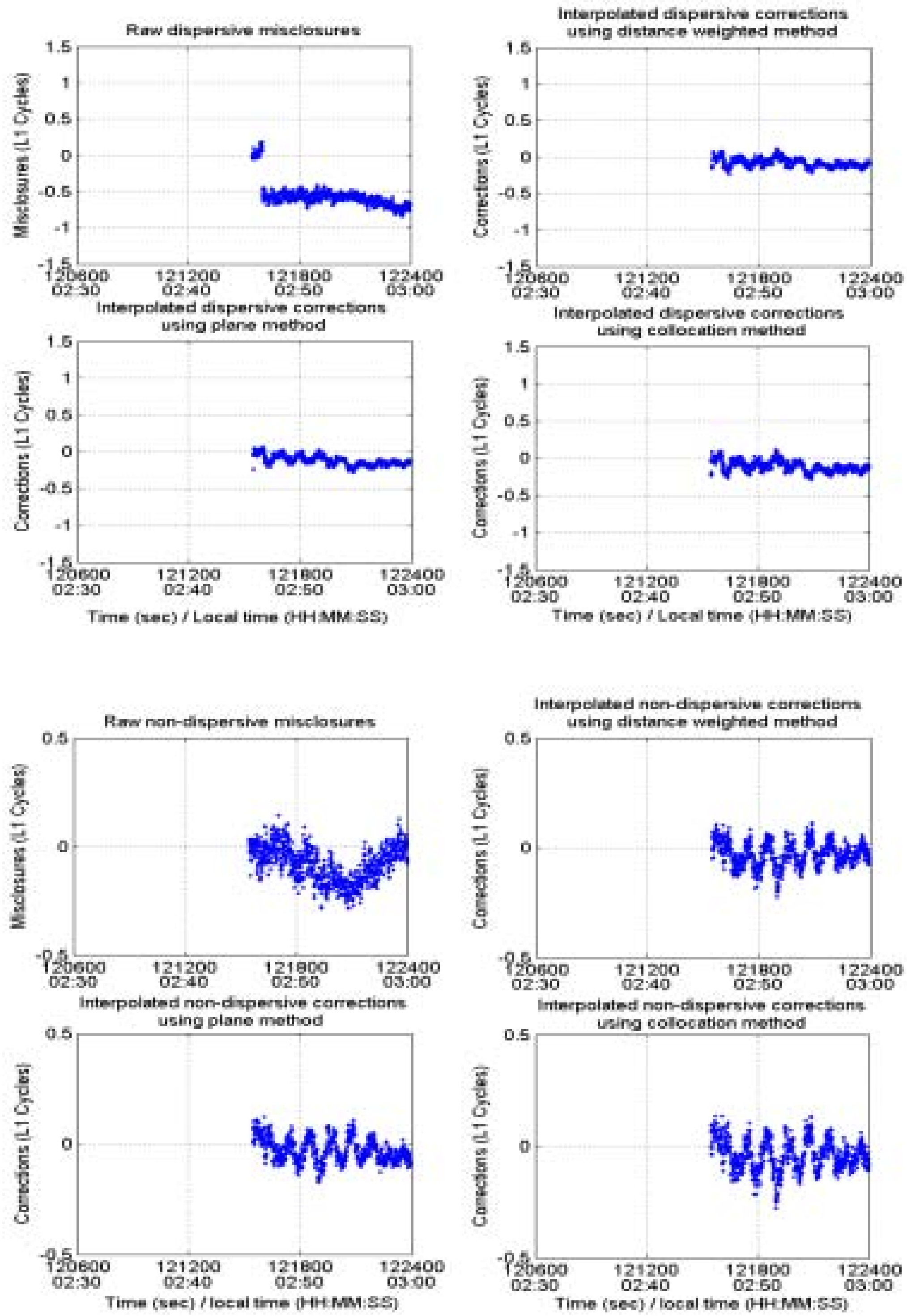

Figure 6.9: PRN4 – PRN 28 (base) “raw” misclosures on baseline IRRI-UOFC and interpolated corrections

6.4.3 Position Domain

GravNav^{TM} 7.01 was used to process the data starting at epoch 115200, 1:00 local time when medium level ionospheric errors were observed.

Twenty-four kilometres from the rover UOFC, the nearest reference station, AIRD, is used for single reference station processing. Three interpolation methods are used in the network approach. The north, east and vertical errors are shown in Figure 6.10 and their accuracies after ambiguity fixing are shown in Table 6.5. The first fixing time for plane and collocation methods is the same, while the distance weighted method takes twenty seconds longer to fix. As compared to the single reference station approach, which takes more than half an hour to fix ambiguities, the network approach exhibits a significant advantage in reducing the convergence time under medium to high ionospheric situations. Even after the single reference station solution is fixed, position errors still increase with time. One possible reason for this is that GravNav^{TM} has fixed the ambiguities incorrectly due to ionospheric effects. All three interpolation methods are effective and demonstrate similar result in positioning accuracy, while collocation method shows slightly better 3D positioning accuracy.

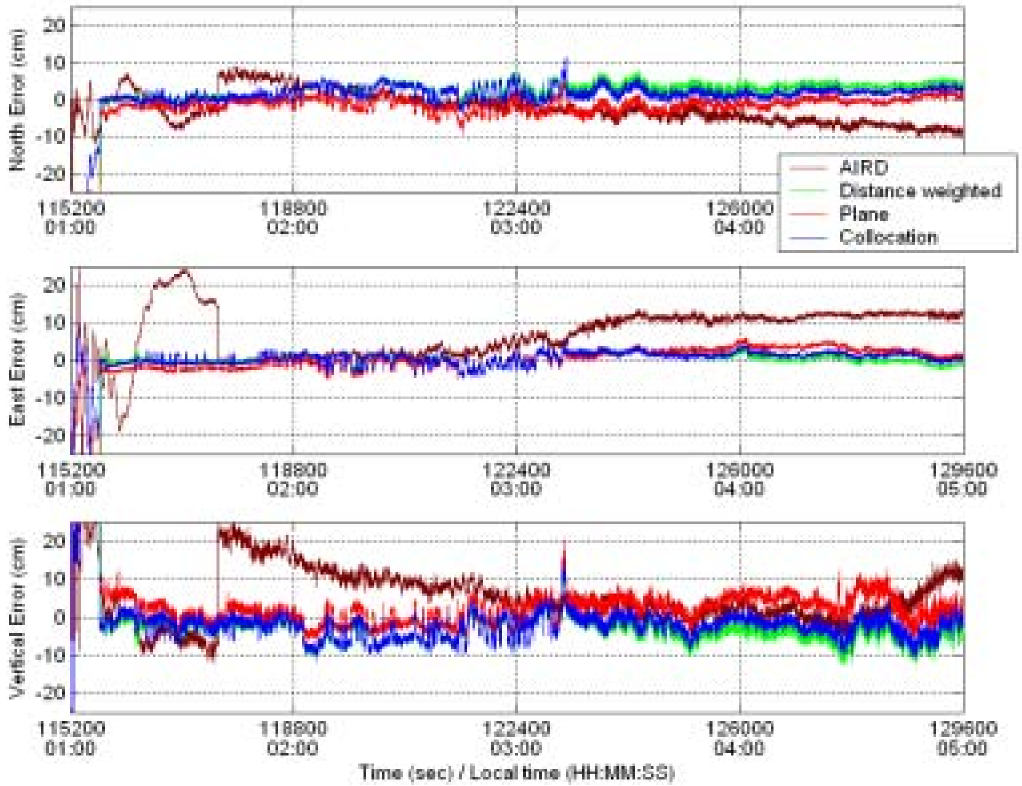

Figure 6.10: North, east and vertical position errors over time for the single reference station and multiple reference station approaches using three interpolation methods.

Table 6.5: RMS position errors of the single reference station approach and multiple reference station approach using three interpolation methods.

Interpolation Methods	**RMS After Fix (cm)**		**First Fixing Time (s)**
AIRD (Single Baseline)	North	4.80	2379
	East	8.55	
	Up	8.73	
	3D	13.13	
Distance weighted	North	3.32	491
	East	1.38	
	Up	4.54	
	3D	5.79	
Plane	North	2.11	470
	East	2.46	
	Up	4.11	
	3D	5.23	
Collocation	North	2.44	470
	East	1.61	
	Up	3.78	
	3D	4.78	

6.5 Test Scenario II

As shown in Table 6.1, the K value stays at a level of 6, starting from local time 23:00, through to 05:00 in Apr 20, 2005. The period from local time 23:00 to 03:00 is selected for analysis in the ambiguity and position domains. Because it is desirable to have one consistent base satellite for the convenience of analysis in observation domain, a shorter period from local time 23:00 to 00:00 is selected and PRN 13 is selected as the base satellite.

6.5.1 Ambiguity Resolution Status

Again, the status of L1 and L2 ambiguities of each baseline in the network from local time 23:00 to 3:00 is shown in Figure 6.11. The percentages of fixed ambiguities associated with each baseline are shown in Table 6.6. It can be seen that the percentages of fixed ambiguities is fairly low, except for baseline IRRI-AIRD. In particular, the success rate of the baseline COCH-BLDM is less than 20% for both L1 and L2 because BLDM suffered frequent cycle slips during this period. Scenario II gives a typical example of a situation when the network software is incapable of resolving many of the ambiguities.

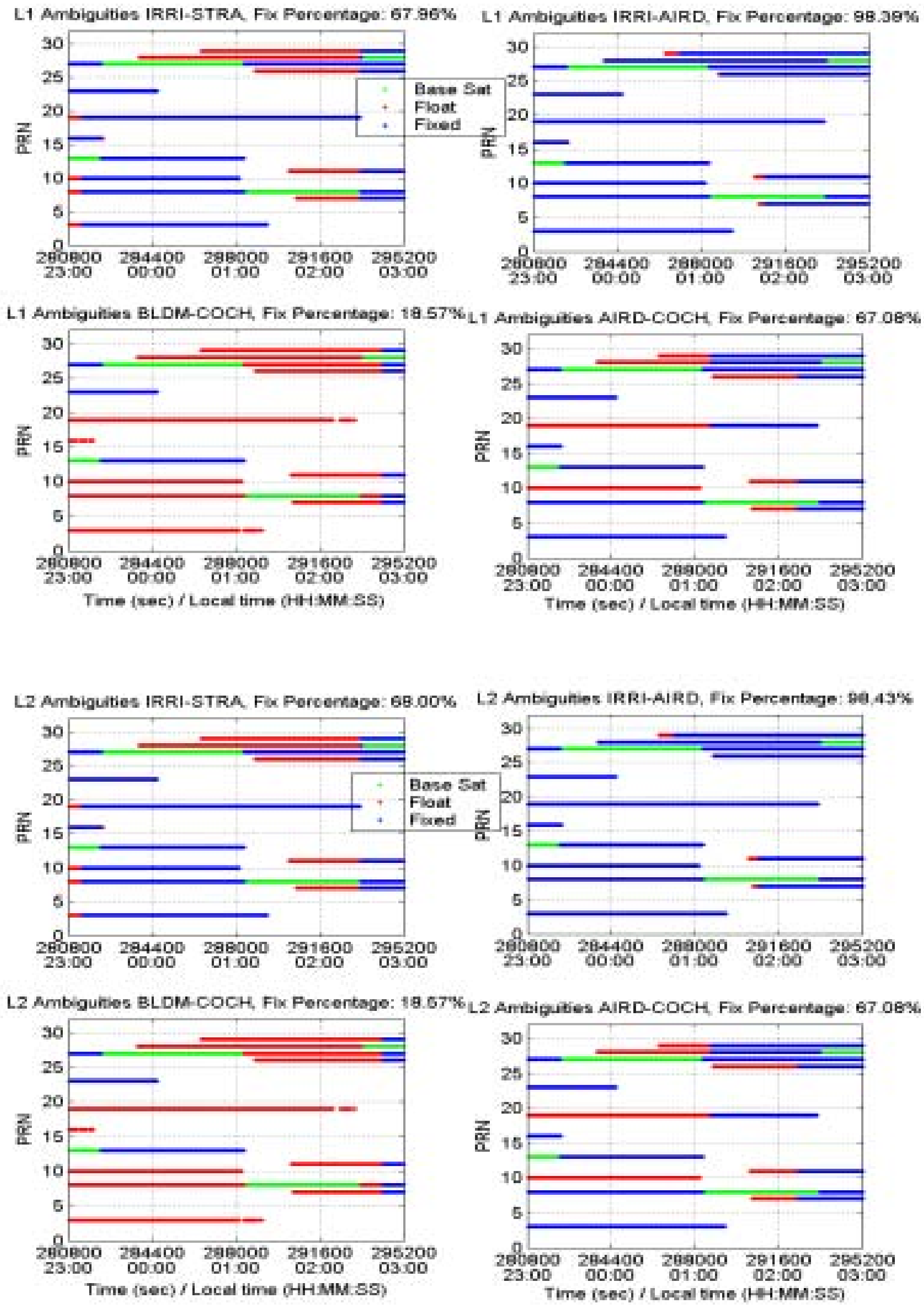

Figure 6.11: L1 & L2 ambiguity resolution status in the network

Table 6.6: Percentage of fixed ambiguities

Baseline	IRRI-STRA	IRRI-AIRD	COCH-BLDM	COCH-AIRD
L1 %	67.96	98.39	18.57	67.08
L2 %	68	98.43	18.57	67.08

6.5.2 Observation Domain

MultiRef™ was run again using the raw observables of IRRI and UOFC. The status of L1 and L2 ambiguities of all satellites is shown in Figure 6.12. In this case, less than 40% of the ambiguities are fixed.

The first one-hour segment of data was selected for analysis in which PRN 13 was chosen as the base satellite. Figure 6.12 shows that only PRN 16 and PRN 23 are fixed throughout this period. The ambiguities of all the other satellites are float or partially float.

Figure 6.12 shows that only PRN 16 and PRN 23 were fixed for baseline IRRI-UOFC. If assuming they are fixed correctly, their misclosures should be equivalent to the true ionospheric and geometric errors. Figure 6.13 shows the dispersive and non-dispersive “raw” and “corrected” misclosures of PRN 23. Similar to scenario I, according to misclosures with and without corrections, an improvement of 70% can be observed with the dispersive components. However, no distinct improvement can be observed with the non-dispersive part since the magnitude of misclosures is as small as 0.1 cycles in this case.

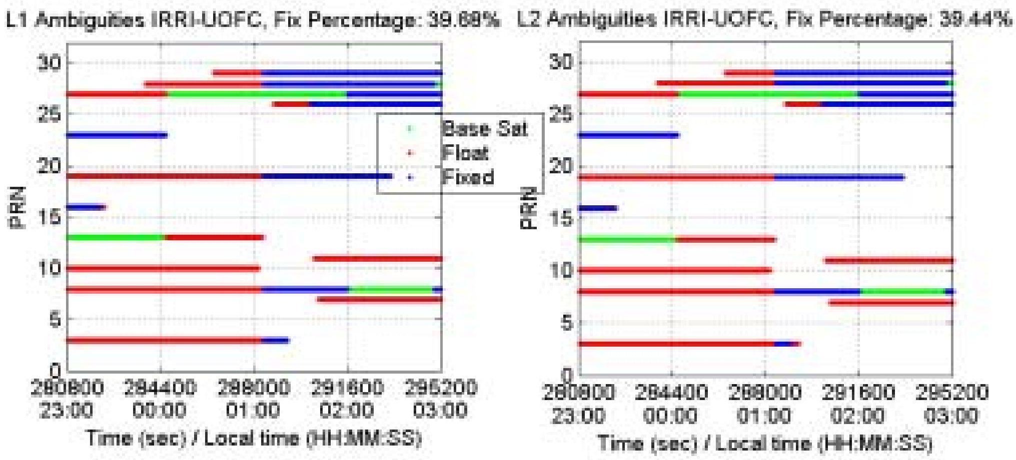

Figure 6.12: L1 and L2 ambiguity resolution between IRRI and UOFC

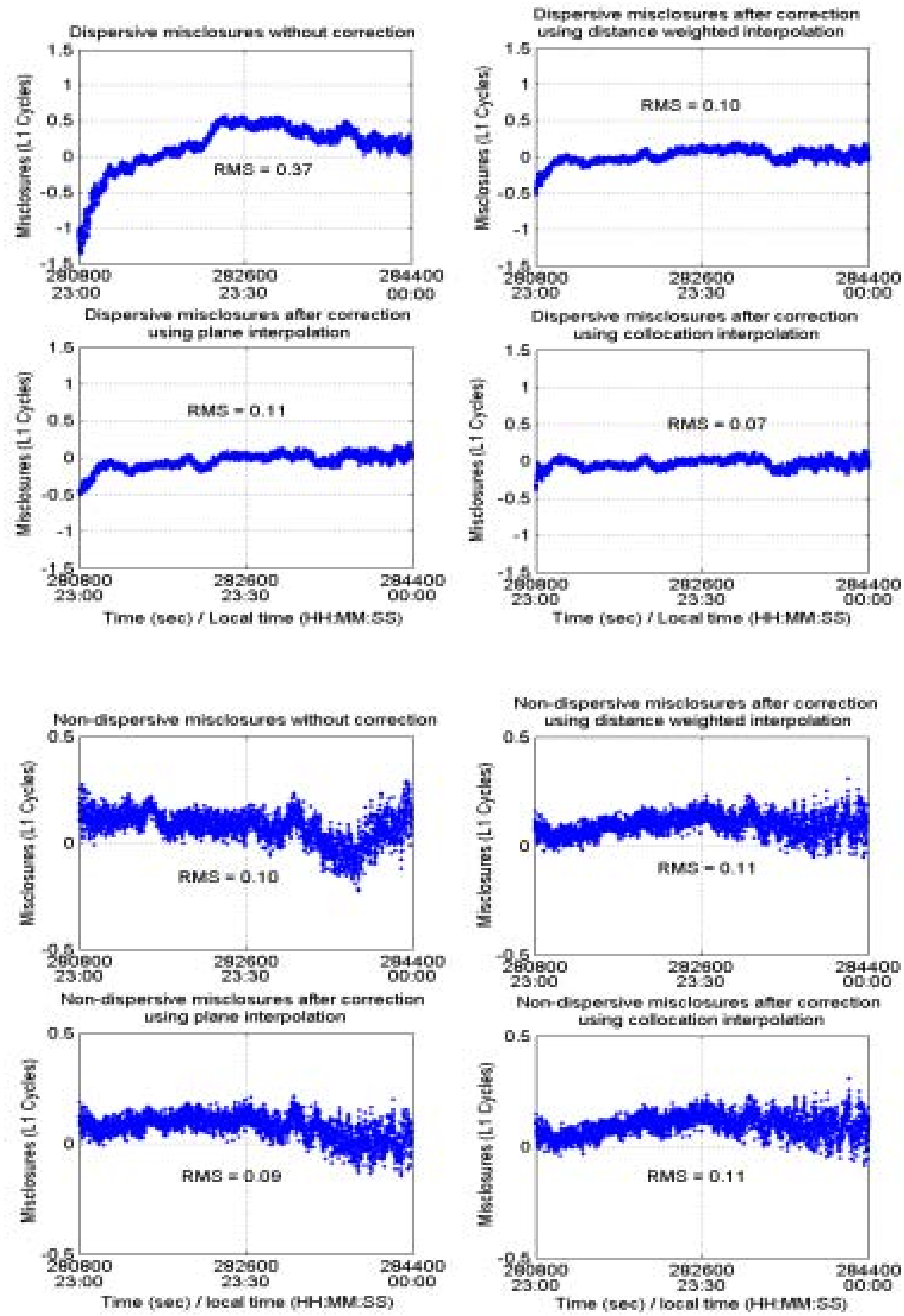

Figure 6.13: PRN 23 – PRN 13 (base) dispersive and non-dispersive "raw" and "corrected" misclosures using distance weighted, plane and collocation interpolation methods

The first plots in Figure 6.14 and Figure 6.15 present the dispersive and non-dispersive misclosures between IRRI and UOFC of all the float satellites, respectively. The other three plots in each figure depict the interpolated corrections generated from the RTCM 3.0 network corrections using the three proposed interpolation methods. Because the ambiguities of these satellites are not resolved, their misclosures do not represent their true ionospheric and geometric errors, as discussed in Section 6.2. The absence of the true value of the relevant errors makes it impossible to evaluate the correctness of the network corrections.

However, it should be noted that, as shown in Figure 6.11, the ambiguities between IRRI, STRA, AIRD and COCH are resolved well, above 60% of the time during the period of interest and MultiRef™ only encounters difficulty to resolve COCH-BLDM ambiguities, in which case the percentage is lower than 20%. Therefore, the network corrections may have the potential to partially cancel out the errors since portions of the corrections are equivalent to the true values. But, it is hard to prove this in the observation domain because the true dispersive/non-dispersive errors cannot be extracted when the ambiguities are not resolved.

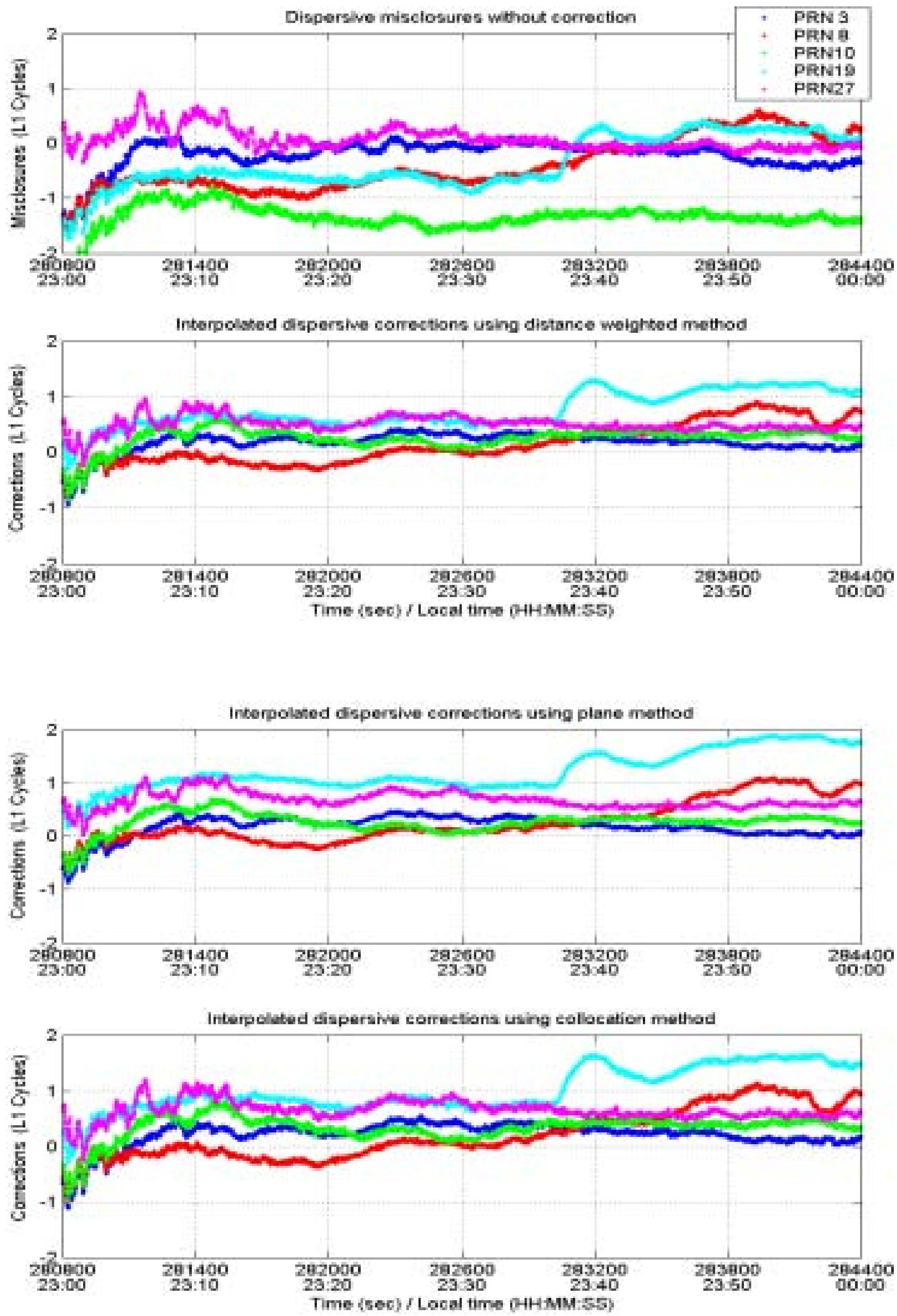

Figure 6.14: Dispersive “raw” misclosures and interpolated corrections using distance weighted, plane and collocation interpolation methods

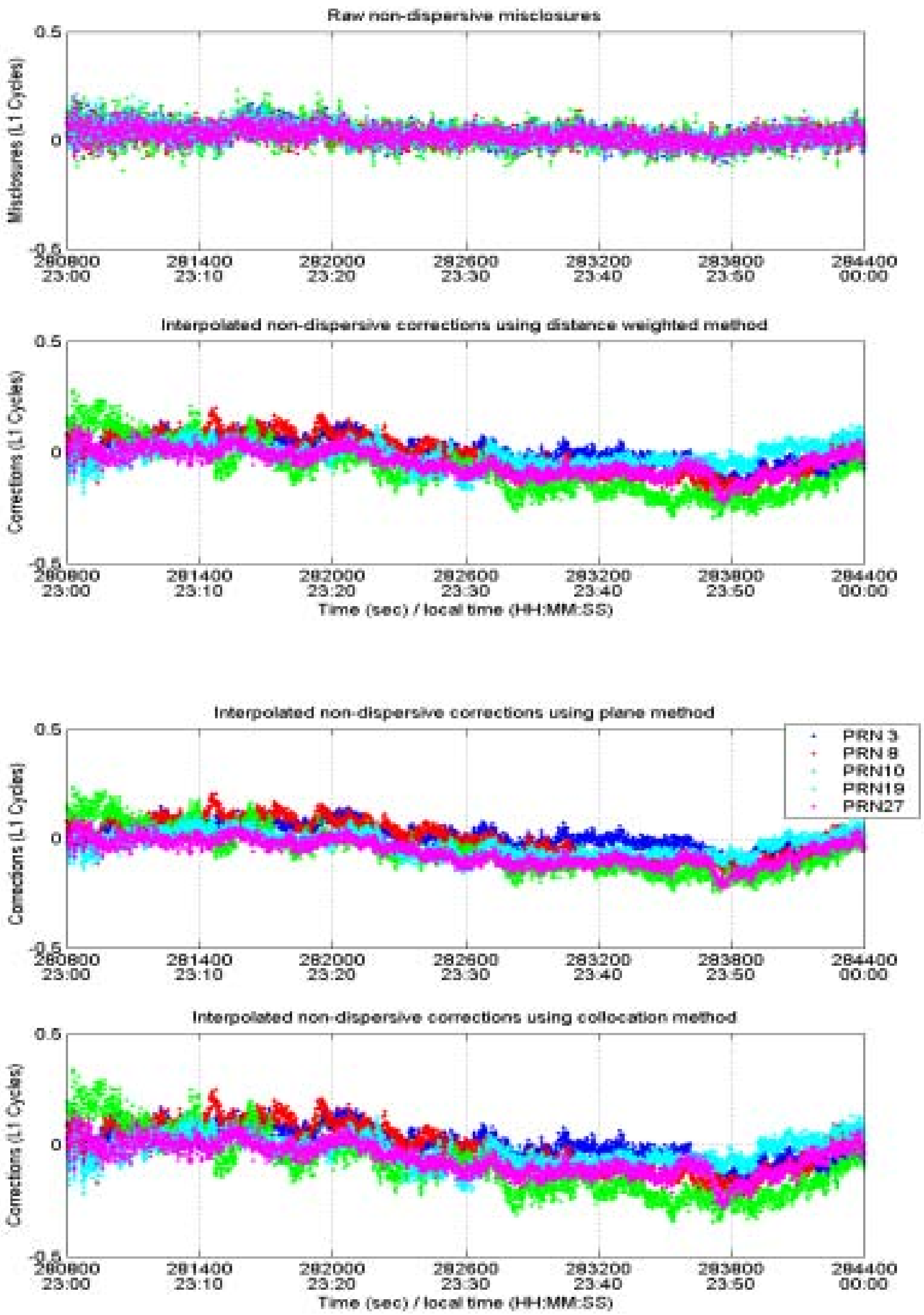

Figure 6.15: Non-dispersive "raw" misclosures and interpolated corrections using distance weighted, plane and collocation interpolation methods

6.5.3 Position Domain

Similarly to Scenario I, the master station (IRRI) raw observables and corrected observables are processed with GravNav^{TM} . GravNav^{TM} started processing at local time 23:00. For comparison, single baseline processing is also carried out using Baseline AIRD-UOFC during the same period, also using GravNav^{TM}.

As discussed in Section 6.5.2, the correctness and appropriateness of "float" corrections cannot be evaluated in the observation domain. Moreover, according to the theoretical analysis given in Section 6.2, the "float" corrections do not reflect the true errors at the physical station point. Consequently, the interpolated corrections are questionable and, hence, the results of employing these corrections are unpredictable. In the following analysis, two GravNav^{TM} runs are reported: one run with Kinematic Ambiguity Resolution (KAR) (GravNav Manual 7.01) enabled and the other with KAR disabled, in which (latter) instance GravNav^{TM} did not attempt to fix the ambiguities and remained in float mode.

First, GravNav^{TM} was run with KAR enabled. The north, east and vertical component errors are shown in Figure 6.16, and the RMS of the positioning errors after ambiguity fixing are shown in Table 6.7. The single station approach fixed the ambiguities at epoch 2814, while the network approach using the distance-weighted interpolation method achieved fix at epoch 1803. The network approach, using plane and collocation interpolation methods, both fixed at epoch 3604, which is later than the single station approach. Moreover, it is obvious from Figure 6.16 that GravNav^{TM} fixed the ambiguities incorrectly using the plane method. A comparison of the RMS of the north, east and vertical errors after the ambiguities are fixed reveals no improvement using the network approach. For this data set, the 3D RMS position errors of the network approach using the distance weighted interpolation method are the best, followed by the single baseline approach and the network approach using the collocation interpolation method. In this case, plane method shows a distinct error in all three dimensions, so the positioning errors of the plane method are not reported in Table 6.7.

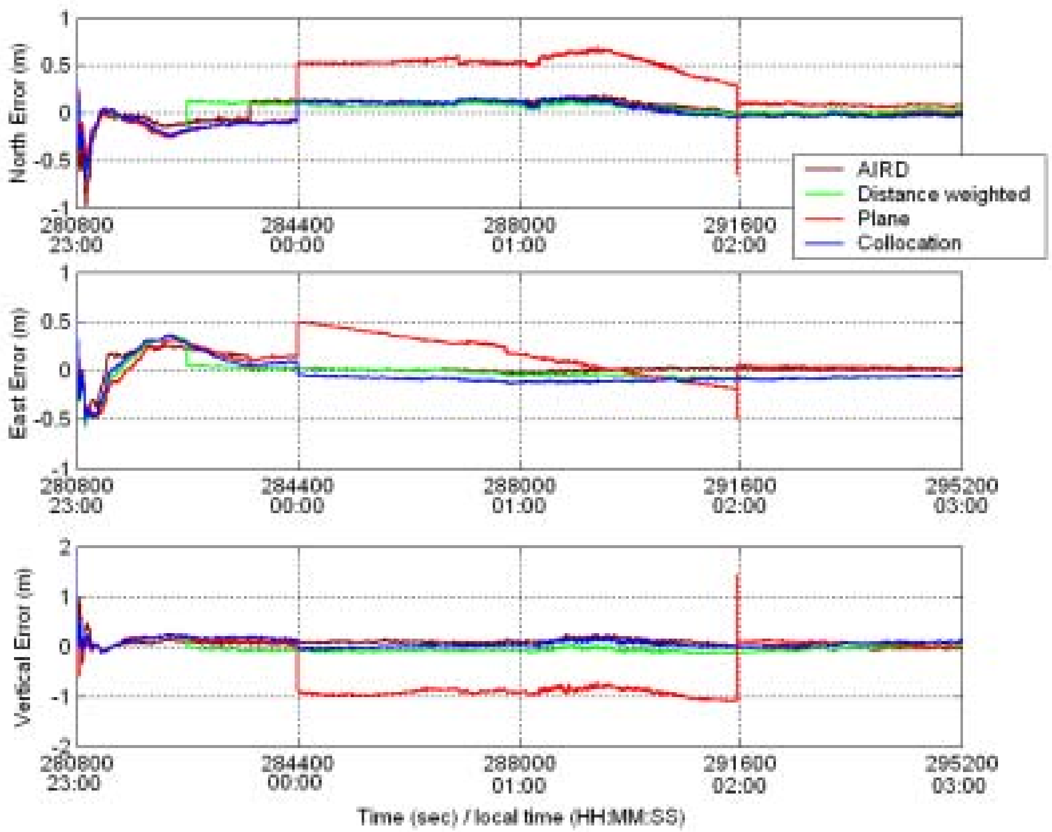

Figure 6.16: North, east and vertical position errors over time for the single reference station approach and multiple reference station approach using three interpolation methods with KAR enabled in GrafNav™

Table 6.7: RMS position errors of the single reference station approach and the network approach using three interpolation methods.

Interpolation Methods	RMS After Fix (cm)		First Fix Time (s)
AIRD (Single Baseline)	North	9.62	2814
	East	1.62	
	Vertical	9.09	
	3D	13.33	
Distance weighted	North	7.25	1803
	East	5.40	
	Vertical	7.71	
	3D	11.88	
Plane	North	N/A	3604
	East	N/A	
	Vertical	N/A	
	3D	N/A	
Collocation	North	8.93	3604
	East	8.72	
	Vertical	5.99	
	3D	13.84	

Again, GravNav^{TM} was run with KAR disabled, i.e. in IF mode. The north, east and vertical errors are shown in Figure 6.17, with their accuracies after one hour shown in Table 6.8. In this run, the network approach using the plane interpolation method performed best in this run, while exhibiting a minimum 3D error of 5.11 cm for the four tests. The distance-weighted interpolation method performed second best, followed by the collocation method and, finally, the single baseline method. Overall, a maximum improvement of 3 cm can be observed using the network approach. A possible reason for this is that the network corrections still minimize the geometric errors to some extent.

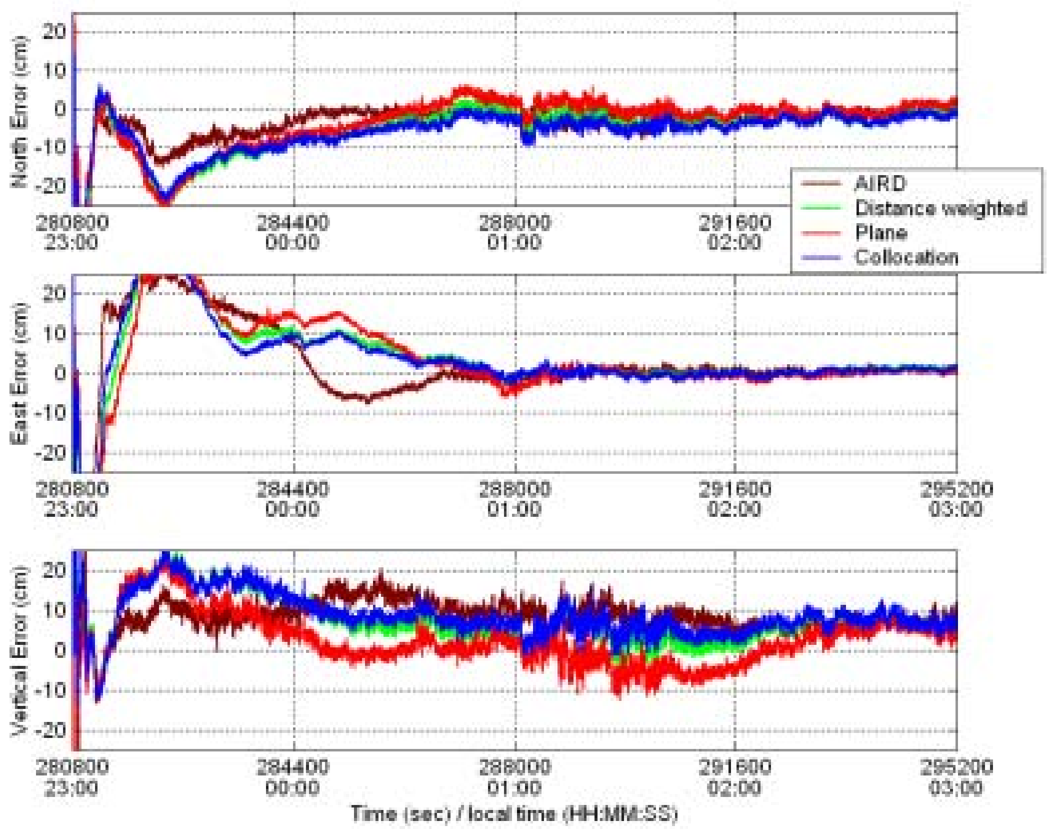

Figure 6.17: North, east and vertical position errors over time for the single reference station approach and multiple reference station approach using three interpolation methods with KAR disabled in GrafNav™

Table 6.8: RMS position errors of the single reference station approach and network approach using three interpolation methods

Interpolation Methods	**RMS After 1 hour (cm)**	
AIRD (Single Baseline)	North	2.64
	East	1.02
	Vertical	7.51
	3D	8.03
Distance weighted	North	3.04
	East	0.92
	Vertical	5.54
	3D	6.39
Plane	North	1.52
	East	1.16
	Vertical	4.74
	3D	5.11
Collocation	North	3.92
	East	0.85
	Vertical	6.74
	3D	7.84

6.6 Conclusions

As shown in scenario I, when the network ambiguities are mostly resolved (fix percentage is 90% or above), the interpolated network corrections effectively minimize the master-rover misclosures by 60% to 80% for the dispersive part, and by 10% to 30% for the non-dispersive part. Among all three interpolation methods, the collocation method shows the highest improvement - 80% in minimizing the dispersive misclosures. In the position domain, the network approach results in many advantages over the single baseline approach in terms of ambiguity fixing speed and positioning accuracy after ambiguities are fixed. All three interpolation methods are effective and demonstrate similar result in positioning accuracy, while collocation method shows slightly better 3D positioning accuracy.

Scenario II, however, offers a different insight. Over the period of interest, the network ambiguities are partially resolved. The network software transmits both the "fixed" and the "float" corrections to the rover. As stated in Section 6.2, the validity of the "float" corrections are questionable. In addition, very few ambiguities can be resolved between IRRI and UOFC; so the true dispersive and non-dispersive effects cannot be obtained and hence no effective analysis can be performed in observation domain. From the results in the position domain, the network approach does not show distinct advantages over the single baseline approach. Moreover, the plane interpolation method even worsens ambiguity fixing and positioning accuracy. When KAR is disabled, namely ambiguity fixing is not tried, the three network methods all show better positioning accuracy than the single baseline method. The improvement of the network solution over the single baseline solution may lie in the "fix" corrections from a portion of the network, which still partially cancel out geometric errors. Another interesting finding of this test is that the IF float strategy (KAR disabled) performs better than the try-to-fix strategy (KAR enabled). This is because the IF float strategy is less sensitive to ionospheric effects and thus the quality of network corrections.

Chapter Seven: Conclusions and Recommendations

This chapter presents the conclusions made in respect of: (i) the high integrity achieved by the new CRC algorithm in RTCM 3.0 compared with the Version 2 (ii) the significance of data compression for network corrections compared with other implementations using RTCM Version 2; and (iii) improvements through network corrections in the measurement and position domains for the above two scenarios using selected data sets. Secondly, some recommendations regarding the applicability of network RTK are given. Finally, some limits of this work are discussed.

7.1 Conclusions

RTCM 3.0 instituted a wide range of efforts to compress the transmitted data. It avoids transmission of very large numbers to decrease the dynamic range of the data. Moreover, it transmits differences only, instead of absolute values to further compress the data. Also, RTCM 3.0 readily incorporates network messages and, through the introduction of the Master-Auxiliary and dispersive and non-dispersive concepts, it demonstrates the potential to reduce the bandwidth by 80%, as compared to RTCM 2.3.

When network ambiguities are resolved above 90% of the time, the majority of the corrections at each station are equivalent to the true value of ionospheric and tropospheric errors. The three interpolation methods proposed herein present similar results when interpolating the network corrections to the location of the rover. Analysis of the sample dataset observed on May 24, 2005 indicates that network corrections can efficiently reduce the master-rover dispersive errors by 60% to 80%, and non-dispersive errors by 10% to 30 %. In all cases, the network approach, regardless of which interpolation method is used, takes less time to fix the ambiguities than does the single reference approach. There is no significant difference among the three interpolation methods in the position domain using the above dataset.

When many of the network ambiguities are not resolved, the "float" corrections at each station depart from the true value of ionospheric and tropospheric errors. Because the true value of ionospheric and tropospheric errors cannot be obtained since the ambiguities between the master station and rover receiver cannot be resolved by MultiRef™, it is difficult to assess the accuracy of the interpolated corrections at the receiver location in the observation domain. Analysis in the position domain shows that the network approach does not show significant advantages over the single baseline approach when KAR is enabled. Moreover, the network approach, using the plane interpolation method, worsens the ambiguity fixing and positioning accuracy when KAR is enabled when using the above dataset. However, when KAR is disabled, the use of the network approach gave superior results. This may lie in the "fix" corrections from portion of the network, which still partially cancel out geometric errors.

Some of the above accuracy performance conclusions are based on the results obtained with only tow data sets. Previous experience has shown that accuracy performance can vary significantly as a function of the network scale, data quality, etc (e.g. Luo 2005, Dao 2005)

7.2 Recommendations

From the two typical scenarios given in Chapter Six, the issue of “float” corrections should be carefully investigated and handled. If possible, the “fix” and “float” corrections should be flagged when they are transmitted to the rover.

When the assumption that all the network ambiguities are resolved correctly holds true, the corrections at each physical station should be equal to the true errors at the respective stations. However, errors may be introduced in the corrections via float ambiguities or incorrect fixed ambiguities, which result in the corrections departing from the true value. If an appropriate indicator associated with the corrections could be transmitted to the rover receiver, it could help the rover receiver to distinguish between the “fix” and “float” corrections. Thus, the rover receiver could take proper measures to deal with the “float” corrections. In RTCM 3.0, although a 2-bit field named the “Ambiguity Status Flag” is defined for each correction, no value is reserved for float indication. For the network service providers who want to utilize “float” corrections to rovers, RTCM 3.0 does not provide a corresponding field. An alternative approach would be to transmit only the “fix” corrections to the rover. However, this introduces availability-related problems. In some cases, the network software can hardly resolve any ambiguities and therefore can generate very few “fix” corrections. The network solution may not be available if less than three satellites are fixed in the network in cases when the ionosphere is active.

Alves *et al* (2005) proposed a geometry-based quality index to monitor network RTK quality. However, the quality index is still based on the assumption that the true errors are determined; i.e., that network ambiguities are correctly resolved. More effort is still needed to indicate the quality of the corrections, and especially the “float” corrections.

As for network RTK and its implementation in RTCM 3.0, there are many more related research avenues that can, and should, be pursued. Some of these include:

1. Analysis of various network configurations. Alves *et al* (2004b) defined topology as different baseline configurations between reference receivers and the rover receiver. Star, line, radial, and shortest baseline topologies are given and investigated in his paper. However, the same limitations apply to Alves *et al* (2004b) as to this research, as previously mentioned, stemming chiefly from the fact that the rover is static. Although, when processing the data, both studies set the mode of the rover to kinematic, some potential problems may remain hidden both as a consequence of, and external to, this choice of rover. Further investigation is warranted using a truly kinematic rover, which could travel from inside the network to outside.

2. The effect of the RTCM 3.0 data updating period. As discussed in Section 4.6, the schedule of RTCM data streams is identical in both data sets, in which the ICPCD

for each station is updated every second, while the GCPCD is updated every two seconds. The following problems need to be addressed: (1) What effects will arise if the updating periods of ICPCD or GCPCD are extended? (2) Because these effects may change with the variance of the ionospheric activity, what are the optimal update rates to be used for the ICPCD and GCPCD with a good balance of data rate and performance under normal conditions?

3. Fortes (2002) applies a Kalman filter to corrections in order to increase their accuracy when the satellites are observed by a portion of the network. During this research, it is observed that discontinuities occur from time to time in the corrections. For example, the correction of one satellite received at one station at a specific epoch cannot be computed because the observations of that satellite at that station at that epoch are not available. However, the corrections before and after that epoch exist. If a Kalman filter could be applied to predict the correction using prior epochs, this would assist in reducing the consequences of discontinuities associated with corrections.

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Appendix A: Single Difference Matrix

RTCM 3.0 network corrections are based on single difference corrections between each one of auxiliary station and master station as given in Chapter Three. For simplicity of the deduction, a single difference matrix, D, is then defined to explicitly show the relation between the absolute and single-differenced values. First, all phase measurement-minus-range-observables from the network are placed into a single vector, L; this consists of one master station, n auxiliary reference stations and p rovers. Because only the single difference data are used in this case, only the measurements of one satellite "i" are listed.

$$L_n = [l_0^i \quad l_{n_1}^i \quad l_{n_2}^i \quad \ldots \quad l_{n_N}^i]^T$$

where 0 denotes the master station; and n_1 to n_N denote all N auxiliary stations. Thus, a single difference matrix D is generated.

$$D_n = \begin{bmatrix} -1 & 1 & 0 & \cdots & 0 \\ -1 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \cdots & \ddots & \vdots \\ -1 & 0 & 0 & \cdots & 1 \end{bmatrix}_{N\times(N+1)}$$

Consequently, $\Delta L_n = DL_n$. This single difference matrix D will be used in deduction to translate the absolute values to single-differenced values.